MONTANA *Department of* REVENUE

Measuring the Quality
of Reappraisal

Residential Property – 2019 Reappraisal

Montana Department of Revenue

September 2019

Executive Summary

This report demonstrates that the 2019 appraisal meets or exceeds the International Association of Assessing Officers (IAAO) standards of appraisal quality in a majority of cases (International Association of Assessing Officers, 2013). The Department of Revenue met the IAAO standard of having a sample appraisal level within 10 percent of market value. The median sample assessment level was 98.2 percent for residential properties and 97.7 percent for commercial properties. The reappraisal also meets uniformity standards on a statewide level for the both types of property being examined. Further, the increases and decreases in appraised values are due to genuine changes of property value and not to faulty reappraisal.

The rest of this report discusses the sales ratio study performed by the Department to evaluate the 2019 appraisal. The first section discusses commonly used sales ratio statistics, followed by a section comparing the most recent residential appraised values to previous appraised values. Statistics for individual regions, select counties, select municipalities, and valuation methods are also reported. The final section is a similar analysis examining commercial properties.

Measuring the Quality of the 2019 Reappraisal

Introduction

The main goal when appraising property is to appraise it at 100% of true market value (15-8-111, MCA). An appraised value represents an estimate of the true market value of property. It is important that these estimates be as accurate as possible. This analysis will provide confidence in the results of the 2019 appraisal.

The reappraisal cycle ending December 31, 2018 is now complete. The Department of Revenue assigned a new appraised value to each Class Four residential and commercial property that replaced the previous two-year cycle's value. The new appraised value represents an estimate of the true market value of the property on January 1, 2018 (using sales up to January 1, 2018). The old appraised value represents an estimate of the true market value using sales up to January 1, 2016.

Most properties saw an appreciation in value since the last reappraisal, however, significant variation in appreciation levels exists in more narrowly defined areas. For these reasons, the Department must provide assurance that the reason for changes in appraised values and the magnitude of the changes are due to the genuine changes of property value and not due to faulty or poor reappraisal performance. Further, because some over appraised properties will have the effect of 'canceling-out' under appraised properties, it is important to also examine the uniformity of the current appraisal cycle.

Measuring the Quality of Reappraisal

The most common method of measuring the performance of property appraisal is a ratio study. Ideally, the ratio study compares the appraised value with the true market value of property. Because market values cannot be directly observed, sales prices usually represent true market values in ratio studies (International Association of Assessing Officers, 2013). A ratio study analyzes the relationship between the assessed value and sale price of property.

$$\text{Sales Ratio} = \frac{\textbf{Appraisal Value}}{\textbf{Sales Price}}$$

The key data element in any sales ratio study is the ratio of assessed value to sale price. To calculate this ratio, divide the assessed value of the property by the sale price of the property. This, of course, assumes that the sale of the property was an arm's-length transaction, and that the sale value is a reliable estimate of true market value. A ratio of less than 1.00 indicates that the property is under appraised. A ratio of greater than 1.00 indicates that the property is over appraised. In the following example, a property with an assessed value of $80,000 that sold for $100,000 has a ratio expressed as .80 or 80 percent.

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                    Assessed Value


        $80,000      =        .8 or 80%   ◄————   Numeric expression of the relationship
        $100,000

                    Sales Price
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Ratio studies measure two primary aspects of appraisal accuracy: level and uniformity.

Appraisal level: Appraisal level refers to the overall level at which properties are assessed. In Montana, the desired assessment level is 100 percent of true market value. The assessed values rarely exactly match the true market values of property. In good appraisal performance, the over appraisals and under appraisals will balance such that the overall appraisal level is close to 100 percent of true market value (Gloudemans, 1999).

Appraisal uniformity: Appraisal uniformity refers to the variation of over appraisals and under appraisals. The degree to which the appraisals differ from true market value is important. In good appraisal performance, the degree to which appraisals differ from true market values is within acceptable standards (Gloudemans, 1999).

There are standard statistical techniques for measuring and analyzing appraisal level and uniformity. Chapter 5 of *Mass Appraisal of Real Property*, published by the International Association of Assessing Officers (IAAO), outlines these measures and techniques (Gloudemans, 1999).

Measures of Appraisal Level

The three most common measures of appraisal level are the median sales ratio, mean sales ratio, and weighted mean sales ratio. Each measure has advantages and disadvantages. It is common practice to compute all three measures (International Association of Assessing Officers, 2013). Comparison of the measures provides useful information about the distributions of the ratios. For example, wide differences among the measures indicate undesirable patterns of appraisal performance. In addition, it is also desirable to calculate the confidence intervals for each of these statistics so that the range of possible values can be determined with a specified degree of confidence (Eckert, Gloudemans, Almy, & International Association of Assessing Officers, 1990).

Median: The median is the middle ratio when all ratios are ordered by magnitude. The median is the most common measure of appraisal level. An advantage of the median is that it is easy to compute and easily understood. By nature, the median is not affected by extreme ratios (International Association of Assessing Officers, 2013) (DeGrouot & Schervish, 2002).

Mean: The mean is the average ratio (the sum of the ratios divided by the number of ratios). Like the median, the mean is easy to compute and understand. However, unlike

the median, the mean is impacted by extreme ratios. The mean is the least used measure of assessment level (International Association of Assessing Officers, 2013) (DeGrouot & Schervish, 2002).

Weighted Mean: The weighted mean is an aggregate ratio (the sum of all the appraised values divided by the sum of all the sales values). The weighted mean is the appropriate measure for estimating the total market value of the population. The weighted mean gives equal weight to each dollar of value in the sample (as opposed to the mean and median, which give equal weight to each parcel) (International Association of Assessing Officers, 2013) (DeGrouot & Schervish, 2002).

Confidence Intervals: When sampling a larger population, it is necessary to be aware of the difference between the attributes of a particular sample and the characteristics of the overall population being sampled. Confidence intervals are a measurement of how likely the sample statistics are representative of the overall population based on the size and variation of the sample. A confidence interval of a sample statistic is a range of values the true population statistics is likely to be between based on a predetermined level of confidence, usually 95 percent confidence level (Eckert, Gloudemans, Almy, & International Association of Assessing Officers, 1990) (DeGrouot & Schervish, 2002).

Measures of Appraisal Uniformity

Part of determining the quality of reappraisal requires measuring uniformity. It is possible for the appraisal level to be good (close to 100 percent), yet still have unfavorable appraisal performance. This occurs when the appraisal is not uniform. Appraisal uniformity can be measured by the frequency distribution of the ratios, standard deviation, and the coefficient of dispersion.

Frequency Distribution: A display of the number of ratios falling within specified intervals. The distribution can be displayed as a table or as a graph. When observing a frequency distribution, a large percentage of the ratios close to the overall level of assessment and distribution symmetry with respect to the overall level of assessment indicate a good level of uniformity (Gloudemans, 1999).

Standard Deviation: The standard deviation is the primary measure of dispersion in scientific research and can be a powerful measure of appraisal uniformity. In a normal distribution, 68 percent of data will be one standard deviation from the mean, 95 percent will be within two standard deviations, and 99 percent will be within three standard deviations (DeGrouot & Schervish, 2002). For example, if a property group has an average mean ratio of 1.01 (101 percent), and a standard deviation of 0.10 (10 percent), it is assumed that in a normally distributed distribution, 68 percent of data will fall between 0.91 (91 percent) and 1.11 (110 percent). Algebraically, the standard deviation can be calculated with the following formula:

$$\sigma = \sqrt{\left(\frac{\sum_{i=1}^{n}(Ratio_i - \overline{Ratio})^2}{n-1}\right)} \times 100$$

In ratio studies, the larger the standard deviation, the wider the range within which a given portion of properties are appraised relative to market value.

Coefficient of Dispersion: The coefficient of dispersion (COD) is the one of the most used measure of uniformity in ratio studies (International Association of Assessing Officers, 2013). The COD is the average absolute deviation expressed as a percentage of the level of assessment and is calculated by dividing the average absolute deviation by the median. The average deviation is calculated by subtracting the median from each ratio, summing the absolute values of the computed differences, and dividing this sum by the number of ratios. For example, a COD of 10% means that the average percent deviation from the median is (+ or -) 10% (Gloudemans, 1999). The COD is expressed algebraically in the following formula:

$$COD = \left(\frac{\left(\frac{\sum_{i=1}^{n}|Ratio_i - Median|}{n}\right)}{Median}\right) \times 100$$

Good appraisal uniformity for residential properties is associated with low CODs of 15% or less for older, heterogeneous areas and 10% for newer, homogeneous areas (Gloudemans, 1999).

Price-Related Differential: The price-related differential (PRD) is a statistic for measuring assessment regressivity or progressivity (Gloudemans, 1999). Assessment regressivity exists if high-value properties are under appraised relative to low-value properties. Conversely, assessment progressivity exists if high-value properties are over appraised relative to low-value properties (Gloudemans, 1999). The PRD is calculated by dividing the mean sales ratio by the weighted mean sales ratio. A PRD greater than 1.00 suggests appraisal regressivity. A PRD less than 1.00 suggests appraisal progressivity. As a general rule, PRDs should range between 0.98 and 1.03 (Gloudemans, 1999).

The following table displays some the IAAO standards for an appraisal being analyzed with a sales ratio analysis (International Association of Assessing Officers, 2013):

Select IAAO Appraisal Standards	
Level of Appraisal	
Min=90%	Max=110%
Coefficient of Dispersion	
Area	Standard
Single Family Residence	5.0 to 15.0
Larger Urban Areas	5.0 to 10.0
Income Producing Property	5.0 to 20.0
Larger Urban Areas	5.0 to 15.0
Vacant Land	5.0 to 20.0
Seasonal and Rural Land	5.0 to 25.0
Price Related Differential	
Min=0.98	Max=1.03

2019 Appraisal-Residential

The Department's Tax Policy and Research unit conducted a study to assess the quality of the recently completed appraisal. The analysis included computing the measures of assessment level and uniformity as discussed previously. Tax Policy and Research calculated these measures on a statewide basis, regional basis, county basis (where a sufficient number of sales existed), a municipality basis (where a sufficient number of sales existed), and for the valuation method used to appraise the property.

The sales values and corresponding appraisal values were extracted from the Department's property valuation information system and provided the data for the analysis. The data set contained 7,028 residential properties that sold from January 1, 2018 to June 30, 2018 and were considered to be valid sales. The Property Assessment Division (PAD) used standard screening processes to determine the validity of sales. Observations that had a sales ratio outside 1.5 times the inter-quartile ranges from the 25th and 75th percentile were dropped, eliminating 580 observations (8.3 percent). This trimming of sales is standard in these types of studies (International Association of Assessing Officers, 2013), and the resulting data set included 6,448 records.

Trimming the sales in this fashion eliminates ratios that are unreasonable. They can be unreasonable for a variety of reasons (International Association of Assessing Officers, 2013):

- the sales price is not accurate

- the assessed value is not accurate

- there is a mistake in the data entry, or

- The nature of the parcel changed between the sale date and assessment date.

In the case that assessment values do not represent market value, these values are likely to be adjusted by informal reviews.

Residential Analysis Results

Statewide Residential Analysis

The overall statewide level of assessment, as measured by the median ratio, is 98.15 percent. It is recommended that the overall level of assessment should be within 10% of market value (Gloudemans, 1999). The upper and lower bounds of this measurement are also with in this range, so we can say with 95 percent accuracy that the appraisal level satisfies this standard.

The statewide coefficient of dispersion is 5.796 for this sample. This value is below 15, and above five, the recommended level IAAO and indicates good appraisal uniformity (Gloudemans, 1999).

To illustrate the improvement in appraisal levels of the ***current market levels***, the Department did similar analysis using the 2017 appraisal values. This is not to say that the 2019 appraisal cycle did a 'better' or 'worse' job relative to the 2017 cycle, only that the 2019 values are generally closer to ***current market values***. This is illustrated by the fact that the prior cycle appraisal levels fall outside of the IAAO standards with a 95 percent confidence level. This trend also indicates the need for a reappraisal.

The following table displays a summary of the ratio statistics using the 2019 appraisal values.

| Residential Ratio Statistics TY17 Values and TY 19 Values		
Number of Sales	**Prior Cycle Values**	**Current Cycle Values**
Total Observations	**7,028**	**7,028**
Used Observations	**6,628**	**6,448**
Measurement of Appraisal Levels		
Upper Bound	*88.24%*	*98.35%*
Median Ratio	**87.84%**	**98.15%**
Lower Bound	*87.42%*	*97.96%*
Upper Bound	*87.62%*	*98.12%*
Mean Ratio	**87.31%**	**97.94%**
Lower Bound	*87.00%*	*97.76%*
Upper Bound	*85.61%*	*97.48%*
Weighted Mean	**84.95%**	**97.09%**
Lower Bound	*84.29%*	*96.71%*
Measurement of Appraisal Uniformity		
Coefficient of Dispersion	**11.467**	**5.796**
Standard Deviation	**12.727**	**7.332**
Price Related Differentials	**1.028**	**1.009**
Range (1.5x Inter Quartile Range)		
Maximum Ratio in the Sample	**123.3%**	**118.0%**
Minimum Ratio in the Sample	**51.6%**	**78.0%**

The statewide appraisal levels all increased relative to the prior cycle values, as would be anticipated in an appreciating market. Additionally, when looking at the appraisal uniformity, the new appraisal levels are much more uniform as measure by the standard deviation and the COD.

The statewide price-related differential for the current cycle is 1.009, which is within the 0.98 to 1.03 range suggested by the IAAO (Gloudemans, 1999).

The frequency distribution of the sales ratios is displayed in Figure 1. The distribution is a tight, symmetrically curved, and centered about the assessment level of 98.15 percent. These characteristics are evidence of good appraisal uniformity and is further supported by a low standard deviation of 7.332. As the distribution shows, the current cycle has a
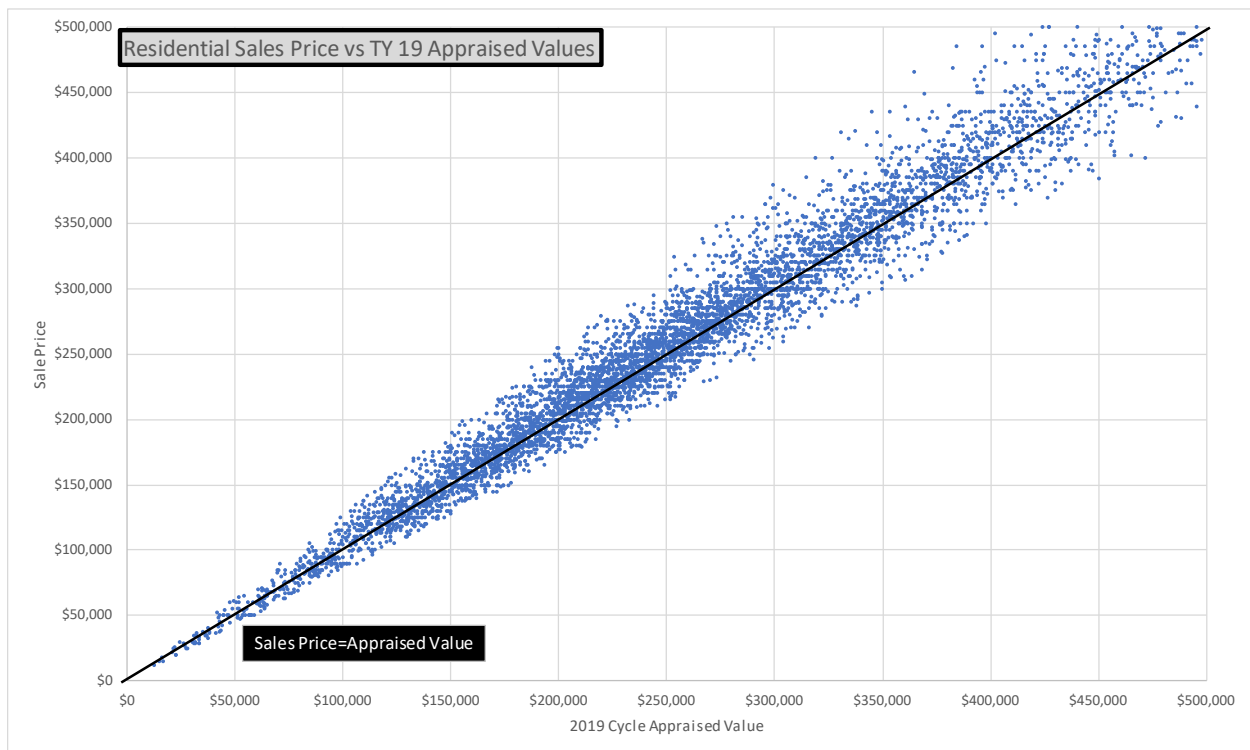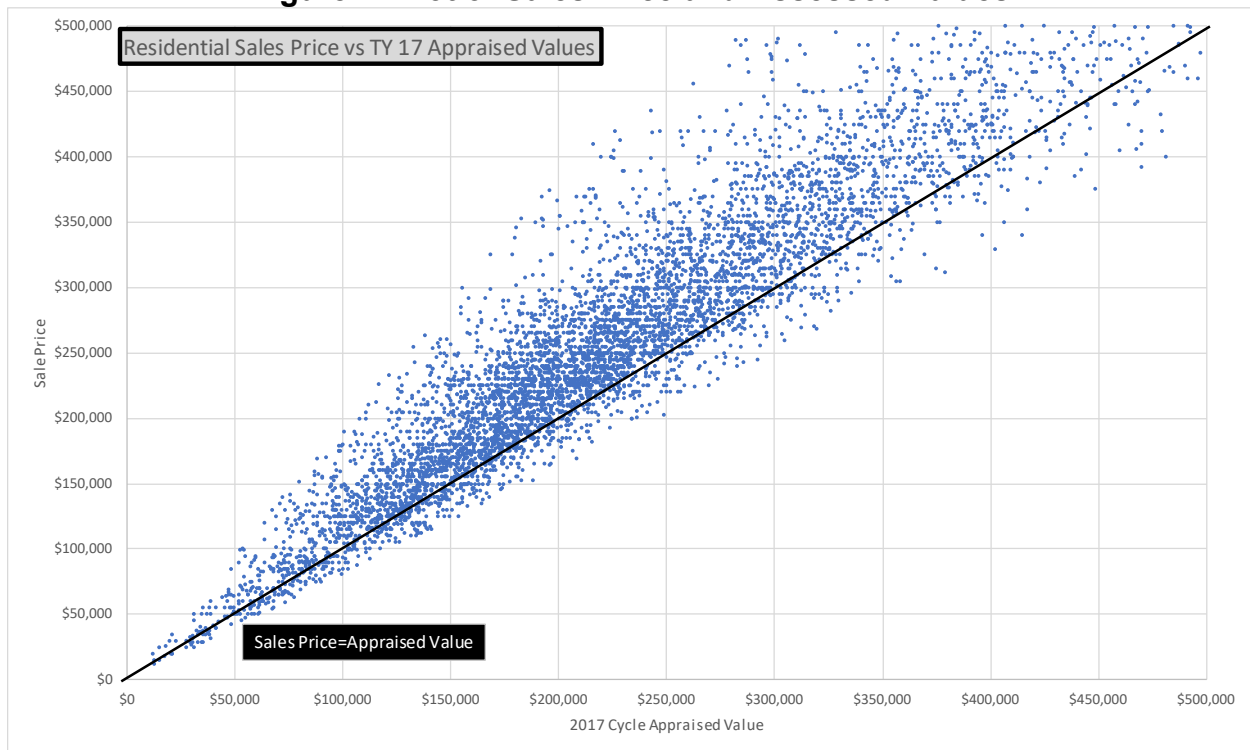
smaller variation and the peak is shifted closer to 1.00. Both trends would be expected given the appraisal levels, standard deviation, and COD values from the previous statewide table.

Figure 1: Sales Ratio Histogram
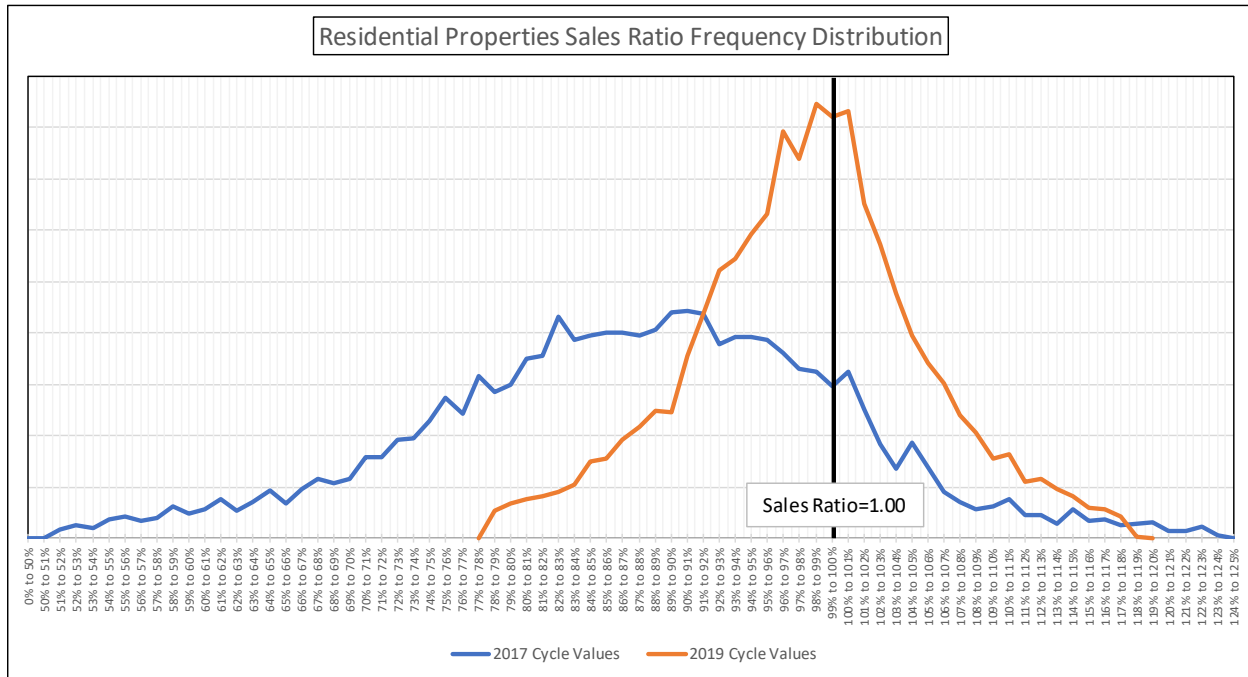
Residential Properties Sales Ratio Frequency Distribution

The following graph shows a (scatter) plot of the relationship between sales prices and assessed values using the **_prior_** appraisal. The next graph has a similar plot of the sales prices but is set against assessed values of the **_current_** appraisal. Each plot has a line where 100 percent of market value is attained, or where sales price equals the assessed value. Values above the line indicate a sales price greater than the assessed value. Similarly, values below the line indicate an assessed value greater than the sales price. As the two graphs show, the prior cycle has significantly more values above the line, and the points are distributed over a large area relative to the current cycle. Again, these trends would be expected given previous statewide table as the scatter plot is a different representation of the same idea.

Figure 2: Plot of Sales Price and Assessed Values

Residential Sales Price vs TY 17 Appraised Values

Sales Price=Appraised Value

SalePrice (y-axis) vs *2017 Cycle Appraised Value* (x-axis)

Residential Sales Price vs TY 19 Appraised Values

Sales Price=Appraised Value

SalePrice (y-axis) vs *2019 Cycle Appraised Value* (x-axis)

Region Analysis-Residential

The Department of Revenue staff calculated reappraisal statistics for the state as a whole, as well as for each of the Department's management regions shown in the following map.
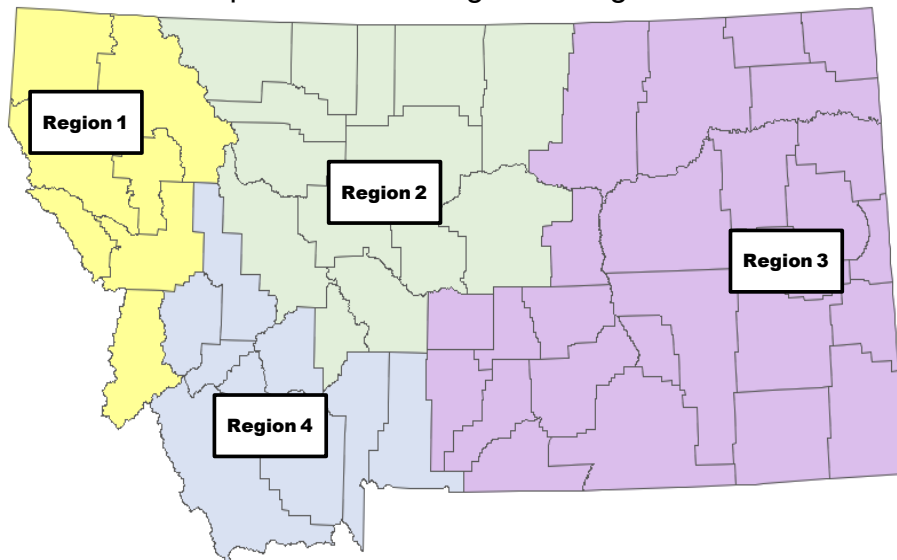

The following tables show the number of verified sales, statistics of central tendencies, and statistics concerning the distribution of the sales assessment ratios for each region. All four regions have median assessment ratios, COD, and PRD values that are within the IAAO recommendation for a quality appraisal (International Association of Assessing Officers, 2013).

Regional Residential Ratio Statistics **TY17 Values and TY 19 Values**								
	Region 1		**Region 2**		**Region 3**		**Region 4**	
Number of Sales	Previous	Current	Previous	Current	Previous	Current	Previous	Current
Total Observations	**2,228**	**2,228**	**1,385**	**1,385**	**1,546**	**1,546**	**1,869**	**1,869**
Used Observations	**2,115**	**2,084**	**1,327**	**1,281**	**1,423**	**1,367**	**1,763**	**1,716**
Measurement of Appraisal Levels								
Upper Bound	*87.15%*	*98.14%*	*92.69%*	*99.10%*	*94.53%*	*99.60%*	*81.21%*	*97.50%*
Median Ratio	**86.63%**	**97.70%**	**91.83%**	**98.74%**	**93.96%**	**99.25%**	**80.67%**	**97.02%**
Lower Bound	*85.99%*	*97.31%*	*90.94%*	*98.23%*	*93.26%*	*98.84%*	*79.96%*	*96.62%*
Upper Bound	*86.91%*	*97.86%*	*91.44%*	*98.87%*	*93.92%*	*99.58%*	*81.49%*	*97.37%*
Mean Ratio	**86.40%**	**97.55%**	**90.79%**	**98.48%**	**93.35%**	**99.19%**	**80.92%**	**97.01%**
Lower Bound	*85.89%*	*97.25%*	*90.15%*	*98.08%*	*92.77%*	*98.81%*	*80.35%*	*96.65%*
Upper Bound	*87.51%*	*97.84%*	*90.71%*	*98.51%*	*93.46%*	*99.21%*	*79.89%*	*96.62%*
Weighted Mean	**86.91%**	**97.50%**	**90.02%**	**98.08%**	**92.86%**	**98.82%**	**78.67%**	**95.76%**
Lower Bound	*86.32%*	*97.16%*	*89.33%*	*97.66%*	*92.25%*	*98.43%*	*77.44%*	*94.91%*
Measurement of Appraisal Uniformity								
Coefficient of Dispersion	**10.843**	**5.591**	**10.030**	**5.706**	**8.618**	**5.558**	**11.537**	**6.165**
Standard Deviation	**11.977**	**7.084**	**11.985**	**7.242**	**11.066**	**7.258**	**12.235**	**7.583**
Price Related Differentials	**0.994**	**1.001**	**1.009**	**1.004**	**1.005**	**1.004**	**1.029**	**1.013**

Although the overall appraisal level went up for the state as a whole, and for each individual region, the change in the assessment level relative to the prior cycle are greatest in Region 1 and Region 4. This would imply that these regions are appreciating in value to a greater degree than properties in Region 2 and Region 3.

The following graphs show the distribution analysis of sales ratios for the four regions using the new appraisal values and the prior cycle appraisal values which is a different way of displaying the same data in the previous table.
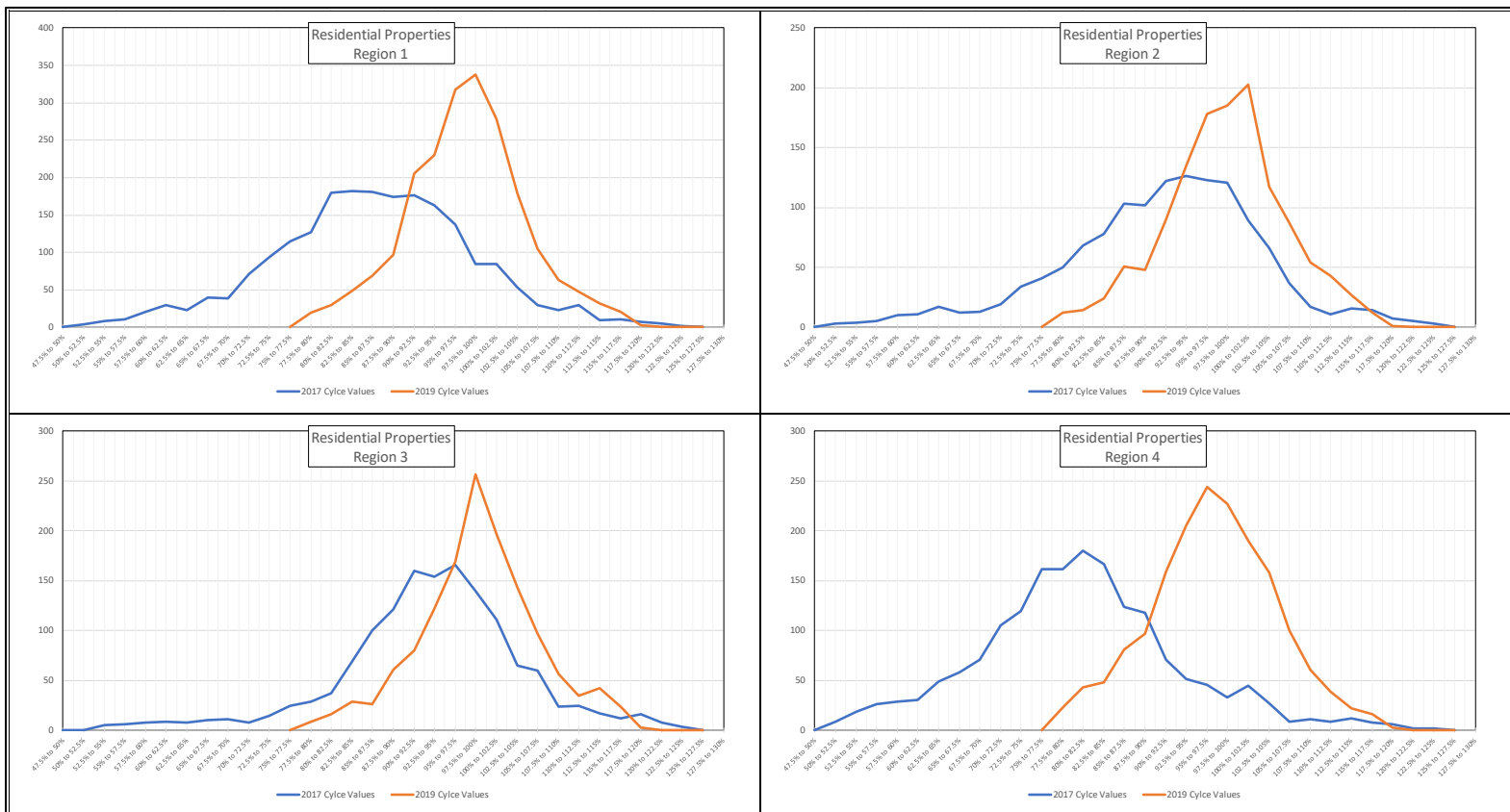
In all for regions, the current distribution has become tighter and more symmetrically centered on one, indicating a good and uniform reappraisal in all four regions. When the prior distribution is centered to the left of the current distribution this is indicative of an appreciating market. It is also worth noting that the shift in the peaks between the previous cycle and the current cycle appear to be greatest in Region 1 and Region 4, indicating further evidence these regions experienced greater appreciation in the current cycle relative to Regions 2 and Region 3.

County Analysis-Residential

There were 23 counties with at least 30 valid sales between January 1 and June 30, 2018. The following table shows the number of verified sales, statistics of central tendencies, and statistics concerning the distribution of the sales assessment ratios.

Select County Wide Residential Ratio Statistics TY 19 Values												
		Measurement of Appraisal Levels									Measurement of Appraisal Uniformity		
County	N	Lower Bound	Median Ratio	Upper Bound	Lower Bound	Mean Ratio	Upper Bound	Lower Bound	Weighted Mean	Upper Bound	Coefficient of Dispersion	Standard Deviation	Price Related Differentials
Beaverhead	51	94.66%	98.62%	101.02%	95.79%	97.97%	100.14%	94.53%	96.65%	98.78%	6.1779	7.7396	1.0136
Broadwater	46	96.92%	97.34%	100.68%	94.30%	96.65%	99.00%	94.49%	97.09%	99.70%	6.7404	7.9076	0.9954
Carbon	49	97.97%	97.19%	101.98%	94.70%	97.74%	100.77%	93.53%	96.90%	100.27%	8.9983	10.5746	1.0087
Cascade	510	99.05%	99.38%	100.05%	98.53%	99.11%	99.70%	98.26%	98.87%	99.49%	5.2190	6.7232	1.0024
Custer	37	97.73%	98.75%	102.72%	96.73%	99.70%	102.67%	96.72%	99.62%	102.52%	7.0198	8.9120	1.0008
Deer Lodge	36	94.93%	98.74%	101.49%	95.80%	98.93%	102.07%	94.15%	97.90%	101.66%	7.3096	9.2712	1.0105
Fergus	79	97.91%	98.49%	101.42%	95.45%	96.87%	98.30%	94.02%	95.84%	97.67%	4.5057	6.3673	1.0108
Flathead	699	96.87%	96.88%	98.19%	96.39%	96.90%	97.42%	96.34%	96.98%	97.61%	5.5771	6.9477	0.9992
Gallatin	1,083	95.14%	97.17%	96.19%	96.74%	97.16%	97.58%	96.46%	97.01%	97.57%	5.7162	7.0736	1.0016
Hill	76	94.91%	97.68%	99.68%	96.57%	98.48%	100.39%	96.19%	97.94%	99.69%	7.2150	8.3649	1.0055
Jefferson	63	95.19%	94.78%	100.58%	92.78%	95.03%	97.29%	92.46%	94.71%	96.97%	7.7439	8.9507	1.0033
Lake	122	97.07%	97.56%	100.06%	95.42%	96.66%	97.91%	95.78%	97.06%	98.35%	5.5249	6.9414	0.9959
Lewis And Clark	467	98.24%	98.39%	99.88%	97.72%	98.38%	99.04%	97.18%	97.87%	98.57%	5.7931	7.2853	1.0052
Lincoln	72	94.90%	98.79%	99.53%	95.85%	97.81%	99.77%	95.91%	97.97%	100.03%	6.7528	8.3283	0.9984
Madison	147	95.29%	94.28%	97.39%	93.04%	94.37%	95.71%	91.30%	93.51%	95.72%	7.0508	8.1915	1.0093
Missoula	812	96.95%	98.26%	97.99%	97.66%	98.12%	98.57%	97.54%	98.04%	98.54%	5.1047	6.6560	1.0008
Park	98	92.58%	96.86%	96.16%	95.24%	96.84%	98.45%	94.98%	96.63%	98.28%	6.4584	8.0050	1.0022
Ravalli	300	96.70%	97.74%	99.15%	96.78%	97.67%	98.56%	96.46%	97.35%	98.24%	6.3038	7.8238	1.0032
Richland	34	97.12%	98.33%	103.94%	94.41%	97.65%	100.88%	93.42%	96.62%	99.83%	7.5127	9.2643	1.0106
Sanders	52	93.61%	99.66%	98.75%	96.51%	98.76%	101.02%	95.97%	98.14%	100.30%	6.2373	8.0913	1.0064
Silver Bow	211	96.01%	98.07%	97.96%	97.12%	98.22%	99.31%	96.43%	97.58%	98.73%	6.5288	8.0860	1.0065
Stillwater	44	94.36%	96.95%	101.69%	94.63%	97.56%	100.49%	93.72%	96.87%	100.02%	7.9490	9.6482	1.0071
Yellowstone	1,053	99.69%	99.47%	100.39%	99.11%	99.48%	99.85%	98.75%	99.14%	99.53%	4.6798	6.1603	1.0035

The level of assessment was calculated for each of these counties. All of the counties have assessment levels (medians) that fall within the IAAO recommended range of 90%-110% (Gloudemans, 1999). Similarly, the PRD and the COD were also calculated for each county, and in all counties, the PRD and COD were inside of the recommended standards by IAAO (Gloudemans, 1999). Carbon county had a COD level of above 10, which would be the standard for large urban areas, but it is below 15, which is the upper standard for areas outside of large urban areas.

Municipality Analysis-Residential

The level of assessment and COD were calculated for the 32 cities and towns (as identified by the properties address) in which there were 30 or more sales. These statistics are listed in the table below.

City	N	Lower Bound	Median Ratio	Upper Bound	Lower Bound	Mean Ratio	Upper Bound	Lower Bound	Weighted Mean	Upper Bound	Coefficient of Dispersion	Standard Deviation	Price Related Differentials
					Measurement of Appraisal Levels						**Measurement of Appraisal Uniformity**		
Anaconda	39	95.35%	98.22%	99.02%	95.43%	98.52%	101.62%	93.62%	97.33%	101.05%	7.6905	9.5497	1.0122
Belgrade	171	92.74%	96.41%	95.89%	95.09%	96.04%	97.00%	94.79%	95.77%	96.74%	5.2112	6.3095	1.0029
Big Sky	72	92.41%	96.84%	96.74%	94.85%	96.67%	98.50%	90.88%	94.37%	97.85%	6.0748	7.7649	1.0244
Bigfork	82	98.40%	98.22%	102.35%	96.19%	97.66%	99.12%	96.08%	97.45%	98.82%	5.0794	6.6721	1.0021
Billings	954	99.94%	99.46%	100.52%	99.13%	99.52%	99.90%	98.74%	99.14%	99.54%	4.5267	5.9929	1.0038
Bozeman	597	94.44%	97.23%	96.18%	96.70%	97.29%	97.88%	96.15%	96.89%	97.62%	5.9493	7.3263	1.0042
Butte	201	95.94%	98.07%	97.75%	97.19%	98.28%	99.37%	96.61%	97.78%	98.95%	6.3452	7.8405	1.0052
Clancy	32	92.05%	95.89%	100.00%	91.67%	95.02%	98.36%	92.00%	95.20%	98.39%	7.8524	9.2823	0.9981
Columbia Falls	74	94.22%	96.68%	100.08%	94.64%	96.48%	98.33%	94.64%	97.96%	101.29%	6.1556	7.9757	0.9849
Corvallis	39	96.39%	98.36%	100.00%	95.17%	97.97%	100.77%	94.97%	97.75%	100.53%	6.7642	8.6325	1.0023
Dillon	49	96.67%	97.81%	100.63%	95.29%	97.37%	99.45%	94.29%	96.41%	98.53%	5.8852	7.2483	1.0100
East Helena	43	98.17%	97.15%	102.71%	95.33%	97.20%	99.08%	94.69%	96.78%	98.87%	5.0663	6.0902	1.0044
Florence	49	94.64%	100.24%	100.95%	98.34%	100.55%	102.75%	97.05%	99.64%	102.23%	5.9936	7.6799	1.0091
Great Falls	476	99.18%	99.45%	100.16%	98.61%	99.20%	99.78%	98.45%	99.04%	99.62%	5.0467	6.5070	1.0016
Hamilton	105	95.26%	97.55%	98.99%	95.78%	97.23%	98.69%	95.46%	96.87%	98.28%	6.0642	7.5310	1.0038
Havre	75	96.40%	97.74%	100.88%	96.72%	98.63%	100.54%	96.31%	98.06%	99.82%	7.1604	8.3134	1.0058
Helena	410	97.52%	98.37%	99.01%	97.62%	98.34%	99.06%	97.06%	97.81%	98.55%	5.8344	7.3759	1.0055
Kalispell	375	95.96%	96.50%	97.70%	95.76%	96.45%	97.14%	95.79%	96.55%	97.32%	5.5804	6.7876	0.9989
Laurel	66	97.27%	100.03%	101.63%	97.93%	99.62%	101.32%	98.15%	99.89%	101.64%	5.5745	6.8977	0.9973
Lewistown	75	96.57%	98.19%	100.71%	95.16%	96.63%	98.10%	93.85%	95.71%	97.57%	4.5906	6.4088	1.0096
Libby	34	92.75%	96.13%	98.43%	94.04%	97.05%	100.06%	93.88%	97.06%	100.25%	7.3997	8.6255	0.9999
Livingston	88	92.63%	96.24%	96.56%	94.79%	96.44%	98.08%	94.48%	96.14%	97.80%	6.3408	7.7663	1.0031
Lolo	70	95.62%	96.53%	99.17%	95.65%	97.52%	99.39%	95.75%	97.75%	99.74%	6.5334	7.8395	0.9976
Manhattan	31	91.42%	95.17%	98.28%	91.88%	94.51%	97.15%	91.17%	94.27%	97.38%	5.3033	7.1825	1.0025
Miles City	37	96.64%	98.75%	102.42%	96.73%	99.70%	102.67%	96.72%	99.62%	102.52%	7.0198	8.9120	1.0008
Missoula	669	96.79%	98.33%	98.20%	97.60%	98.10%	98.59%	97.47%	98.02%	98.56%	4.9467	6.5306	1.0008
Polson	63	95.62%	97.12%	100.00%	95.17%	97.03%	98.89%	95.48%	97.62%	99.75%	5.9669	7.3984	0.9940
Stevensville	87	94.25%	96.74%	97.81%	95.17%	96.78%	98.39%	95.23%	96.75%	98.27%	6.1935	7.5604	1.0003
Three Forks	35	92.06%	95.66%	98.48%	93.73%	96.32%	98.92%	93.17%	95.98%	98.79%	6.8563	7.5530	1.0035
Townsend	32	94.36%	97.81%	103.54%	95.49%	97.97%	100.45%	95.53%	98.47%	101.40%	5.5746	6.8685	0.9950
Whitefish	126	96.42%	98.13%	99.15%	96.76%	97.88%	98.99%	95.54%	96.81%	98.08%	4.7466	6.3266	1.0110

Table title: **Select City Residential Ratio Statistics for TY 19 Values**

All areas have medians in the recommended range (i.e. within 10 percent) (Gloudemans, 1999). The COD values are also all less than the IAAO Standards. However, there are four cities (Billings, Lewistown, Missoula, and Whitefish) where the COD is less than five, the lower bound for the IAAO standard (International Association of Assessing Officers, 2013). This may indicate that the sample is not representative of the overall population, in which case the appraisal is not necessarily bad, but rather may not be as uniform as the COD indicates. Another reason for low COD levels could be that the markets are extremely homogenous and stable. In all four of the cities, the COD is only slightly below the 5.0 A COD lower than 5.0 means some sales chasing may have occurred because it represents too accurate of a value, more than likely sales chasing did occur, but it is probably more likely to be too few and non-representative samples and as long as the representative features of the sales are applied to the subject properties it means accuracy is attained. Further, IAAO standard and is improved relative to the COD using the prior cycle's assessment levels.

Valuation Method-Residential

As an additional check on the quality of the 2019 appraisal, it is helpful to examine sales ratio characteristics based on the method in which properties were appraised. The two primary approaches to valuing residential property are a market-based approach and a cost-based approach. In addition, the Department has the option to override these two values for a variety of reasons. The sales ratio summary statistics for these methods of valuation are presented in the following table.

Residential Ratio Statistics TY 19 Values by Valuation Method			
Number of Sales	**Market**	**Cost**	**Other**
Total Observations	**6,089**	**872**	**67**
Used Observations	**5,733**	**653**	**62**
Measurement of Appraisal Levels			
Upper Bound	98.30%	98.90%	100.63%
Median Ratio	**98.11%**	**98.19%**	**100.00%**
Lower Bound	97.87%	97.39%	100.00%
Upper Bound	98.16%	97.98%	102.98%
Mean Ratio	**97.97%**	**97.31%**	**101.38%**
Lower Bound	97.79%	96.64%	99.78%
Upper Bound	97.51%	97.21%	104.31%
Weighted Mean	**97.09%**	**96.32%**	**101.99%**
Lower Bound	96.68%	95.43%	99.66%
Measurement of Appraisal Uniformity			
Coefficient of Dispersion	**5.6643**	**7.0109**	**4.1508**
Standard Deviation	**7.1561**	**8.7228**	**6.2843**
Price Related Differentials	**1.0090**	**1.0103**	**0.9940**

As the table shows, the three valuation methods are all within the appraisal level standards established IAAO (International Association of Assessing Officers, 2013). Properties whose values were not derived using the cost or income approach had a higher sales ratio by a statistically significant margin, indicating a higher appraisal relative to their sales price. However, this is likely due to the sample selection issues, as these properties may not be representative of the overall population and have an 'other' valuation method as a result. IAAO states that if the different stratification groups have appraisal levels with in five percentage points of the overall appraisal levels, the appraisal is still considered valid (International Association of Assessing Officers, 2013). Additionally, the three groups have COV levels below the standards set by IAAO indicating good uniformity (International Association of Assessing Officers, 2013). Again, the 'other' category has a COD level below the standard set by IAAO, which may indicate that the appraisals are

'too uniform' but this is also likely a byproduct of these properties having a non-standard valuation in the first place. All valuation methods also have acceptable PRDs, indicating that there is not an abnormal level of regressivity as a result of the valuation method.

Conclusion-Residential

Based on widely recognized norms and standards, the 2019 appraisal is generally of high quality, as evidenced by this study. The goal of having a sample appraisal level within 10 percent of market value is met (International Association of Assessing Officers, 2013). The sample assessment level of 98.2 percent is within 2.0 percent of market value.

The reappraisal also meets uniformity standards, as evidenced by the coefficients of dispersion and the price-related differential. The statewide COD of 5.8 is within the accepted range of 5.0 to 15.0, with the lower number reflecting greater accuracy. The PRD of 1.009 is also between the IAAO recommended 0.98 and 1.03 standard (International Association of Assessing Officers, 2013).

Statewide, the ratio distribution of the old and new values seems to indicate appreciation in most areas of the state, with some areas appreciating more than others as evidence by the shift in median sales ratio from the old appraisal values to the new appraisal values.

2019 Appraisal-Commercial

Similar to residential properties, prices seemed to increase in a majority of areas for commercial property. However, increases seemed to be less pronounced relative to the residential properties. Also like residential properties, there seems to be variation in the application of property in different areas of the state. Therefore, the Department must provide assurance that the reason for increases or decrease in appraised values is due to the genuine changes in property value and not due to faulty or poor reappraisal performance.

The sales that occurred were verified by PAD to determine if the sales were usable for valuation purposes. This includes making sure that the sale price is representative of only the market value of real property and insuring that the sales are arms-length transactions.

Oftentimes, sales prices for commercial property include the real property and also the business interest or personal property located inside that property. For example, a gas station may sell for $250,000, but the land could be purchased for $75,000 and the building could be built for $50,000. The cost approach to valuation would value the property at $125,000. The other $125,000 in the sale price is for the established business and personal property (like the gas pumps and the signs). When this is the case, the sale price is not a valid indicator of the market value of real property, but instead represents the market value of the entire business, including the personal property.

Single-family residential property is rarely purchased for anything other than to provide housing. This generally means that there is significantly less distortion in the residential sales price as a result of business interests or personal property, as may be the case in commercial sales.

Another criterion for a sales ratio analysis is for the properties that sell to be representative of all properties being evaluated. In this case, the Department wants to determine if the reappraisal of all commercial properties is accurate. So, the commercial sales must be representative of the commercial properties in the state. This means that the distribution in terms of geography, use, and value of the properties that sell is representative of all commercial properties in the state. Some types of properties only have a very specific use, and there may be only one or two properties of its kind in the state. It is unlikely that these properties sell in any given year, so it is hard to use sales to verify the assessed values on these types of properties. The more sales that occur, the more likely that the sample of sales is representative of the universe of properties.

Even if the sales are not representative of the universe of commercial properties, confidence intervals can be calculated and used to evaluate appraisal quality. A confidence interval determines the range that the true assessment ratio is between. This acknowledges that there may be some variation between the universe and the sample. The use of confidence intervals can also make up for having fewer sales.

Because of limited sales and the complexity of commercial real estate markets, assessing the quality of the appraisal for commercial property is more difficult than assessing the quality of reappraisal for residential property. The quality of commercial reappraisal includes confidence intervals and hypothesis testing because of fewer commercial sales and a more complex commercial market. Statistical tools and tests can then be used to overcome some of the challenges in validating the commercial mass appraisal.

The results for commercial property are not necessarily directly comparable to the results presented for residential property but the two are related.   In acknowledging the complexity of mass appraisal for commercial property, the IAAO has different standards for assessment level and uniformity for commercial and residential property (International Association of Assessing Officers, 2013).

Data-Commercial

The sale prices and corresponding assessment values were extracted from the Department's property information valuation system and provided the data for this analysis. The data set contained 592 commercial properties that sold from October 1, 2017 to June 30th, 2018 that the PAD considered valid sales. The PAD used standard screening processes to determine the validity of sales. This screening is meant to ensure that the sales price represents the market value of the real property.   The screening eliminated sales where the sales price represents the market value of the real property and personal property or an established business.

Ideally, there would be enough sales in the first half of 2018, and sales from the prior year are not needed. Sales before January 1, 2018 were used in the models to determine

assessment value, and therefore the assessed values are not strictly independent of the sales prices. The sales after January 1, 2018 are independent and would be the preferred measure of market value, given enough data.

The assessment ratios for properties that sold in the fourth quarter of 2017 were compared to properties that sold in the first half of 2018. The mean and median assessment ratio were similar, and a t-test indicated there was no statistical difference between the mean assessment levels or the weighted mean of the two groups. Therefore, the full time period's worth of sales can be used to estimate the assessment level for all commercial and industrial properties.

In a valid sales ratio, the properties that sold are representative of all the commercial property in the state. To test this hypothesis, a t-statistic was calculated. The null hypothesis is that the two groups of commercial properties have the same mean assessed value in TY 2019. The t-statistic indicated that we cannot reject the null hypothesis at the 95th percent confidence level. In other words, the properties that sold have similar assessed value as properties that did not sell, indicating that they are similar.

Observations that had a sales ratio outside 1.5 times the inter-quartile ranges from the 25th and 75th percentile were dropped, eliminating 86 observations (14.5 percent). This trimming of sales is standard in these types of studies (International Association of Assessing Officers, 2013), and the resulting data set included 506 records.

Trimming the sales in this fashion eliminates ratios that are unreasonable. They can be unreasonable for a variety of reasons (Gloudemans, 1999):

- the sales price is not accurate

- the assessed value is not accurate

- there is a mistake in the data entry, or

- The nature of the parcel changed between the sale date and assessment date.

In the case that assessment values do not represent market value, these values are likely to be adjusted by informal reviews.

Commercial Results

Statewide Commercial Analysis
Similar to the residential analysis, in order to illustrate the improvement in appraisal levels of the _**current market levels**_, the Department did similar analysis using the 2017 appraisal values. This is not to say that the 2019 appraisal cycle was 'better' or 'worse' relative to the 2017 cycle, only that the 2019 values are generally closer to _**current market values**_. This is illustrated by the fact that the prior cycle appraisal levels are much closer to (or entirely outside of) the IAAO standards with a 95 percent confidence level. This is also an indication of the need for a reappraisal (Gloudemans, 1999).

The following table displays a summary of the ratio statistics using the 2019 appraisal values.

Commercial Ratio Statistics TY17 Values and TY 19 Values		
Number of Sales	**Prior Cycle Values**	**Current Cycle Values**
Total Observations	592	592
Used Observations	552	506
Measurement of Appraisal Levels		
Upper Bound	*90.70%*	*98.74%*
Median Ratio	**88.00%**	**97.71%**
Lower Bound	*84.63%*	*96.56%*
Upper Bound	*87.24%*	*97.70%*
Mean Ratio	**85.44%**	**96.49%**
Lower Bound	*83.65%*	*95.28%*
Upper Bound	*81.32%*	*95.11%*
Weighted Mean	**77.84%**	**92.40%**
Lower Bound	*74.35%*	*89.70%*
Measurement of Appraisal Uniformity		
Coefficient of Dispersion	19.581	10.520
Standard Deviation	21.467	13.864
Price Related Differentials	1.098	1.044
Range (1.5x Inter Quartile Range)		
Maximum Ratio in the Sample	146.8%	133.4%
Minimum Ratio in the Sample	30.6%	61.2%

The statewide overall level of assessment, as measured by the median ratio, is. 97.71 percent. The mean for commercial properties in 2019 was 96.49 while the weighted mean was 92.4 percent. Although all three measures are less than 100 percent by a statistically significant margin, all three statistics are within the IAAO standard of being within 10 percent of the target of 100 percent (International Association of Assessing Officers, 2013).

The measures of uniformity show that the coefficient of dispersion is also with in the acceptable IAAO range of five to 20, indicating the 2019 appraisal had good uniformity for commercial properties (International Association of Assessing Officers, 2013). The PRD is 1.044 which is slightly above the IAAO standard of 1.03. The PRD statistic is much more sensitive to extreme values and higher priced properties, and because of the nature and complexity of commercial properties, a PRD value outside of the IAAO

standard my not be as important as in the case of residential properties (International Association of Assessing Officers, 2013).

The following graph shows the distribution of assessment ratios for the new, 2019 values and the old, 2017 values. Ideally, the distribution would show a tight, symmetrical distribution centered around 1.0. Because the commercial properties have more variation, the line is not as smooth relative to the residential distribution. This is the nature of commercial property assessment studies because there are generally fewer sales and there are more factors in determining the value of commercial property. As the graph shows, there are clearly more properties with an assessment level around 1.0 in the current cycle (the peak is much higher). It is also clear from the graph that the assessment ratios for the prior cycle's values are much less uniform, as the peak is lower, and the tails are wider. Unlike the residential histogram, the shift to the right in the peaks of the distribution is not as pronounced. In the statewide table shown earlier, the median value went up relative to the prior appraisal, and this is because more of the distribution from the prior appraisal is to the left of the peak.
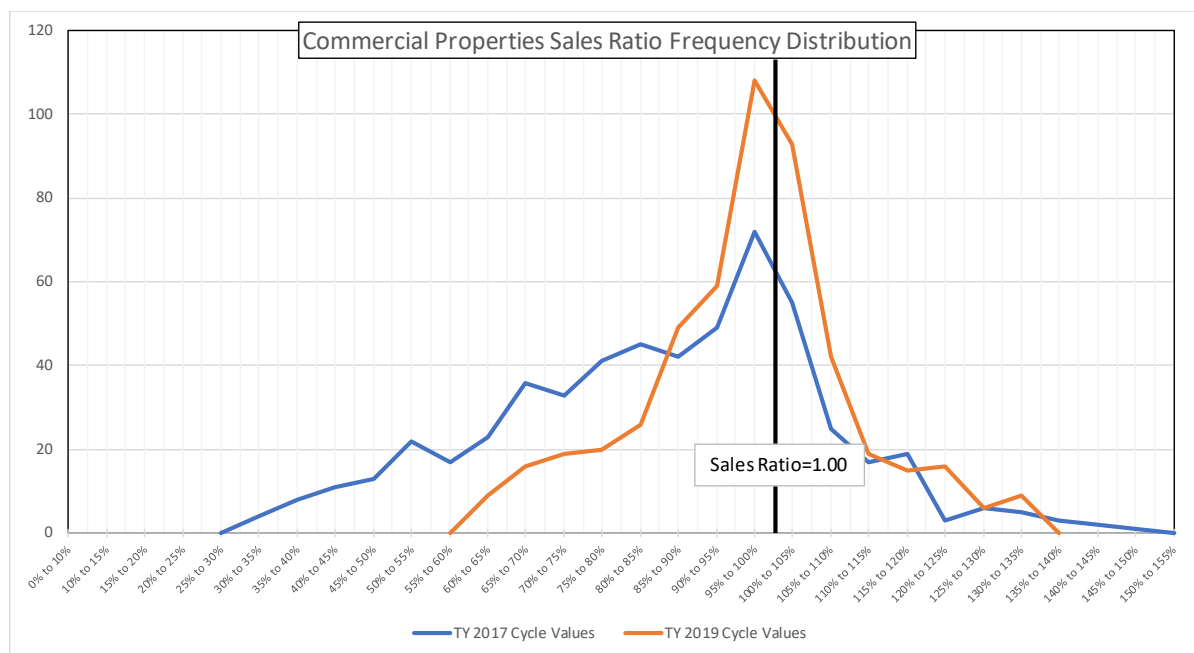
The following graph shows a (scatter) plot of the relationship between sales prices and assessed values using the *prior* appraisal. The next graph has a similar plot of the sales prices but is set against assessed values of the *current* appraisal. Each plot has a line where 100 percent of market value is attained, or where sales price equals the assessed value. Values above the line indicate a sales price greater than the assessed value and values below the line indicate an assessed value greater than the sales price. As the two graphs show, the prior cycle has significantly more values above the line, and the points are distributed over a large area relative to the current cycle. Again, these trends would be expected given previous statewide table. Unlike the residential scatter plot, the

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difference between the two scatter plots is not as dramatic given fewer sales, and a more complicated and variable market for commercial properties.

Commercial Sales Price vs TY 17 Appraised Values

Commercial Sales Price vs TY 19 Appraised Values
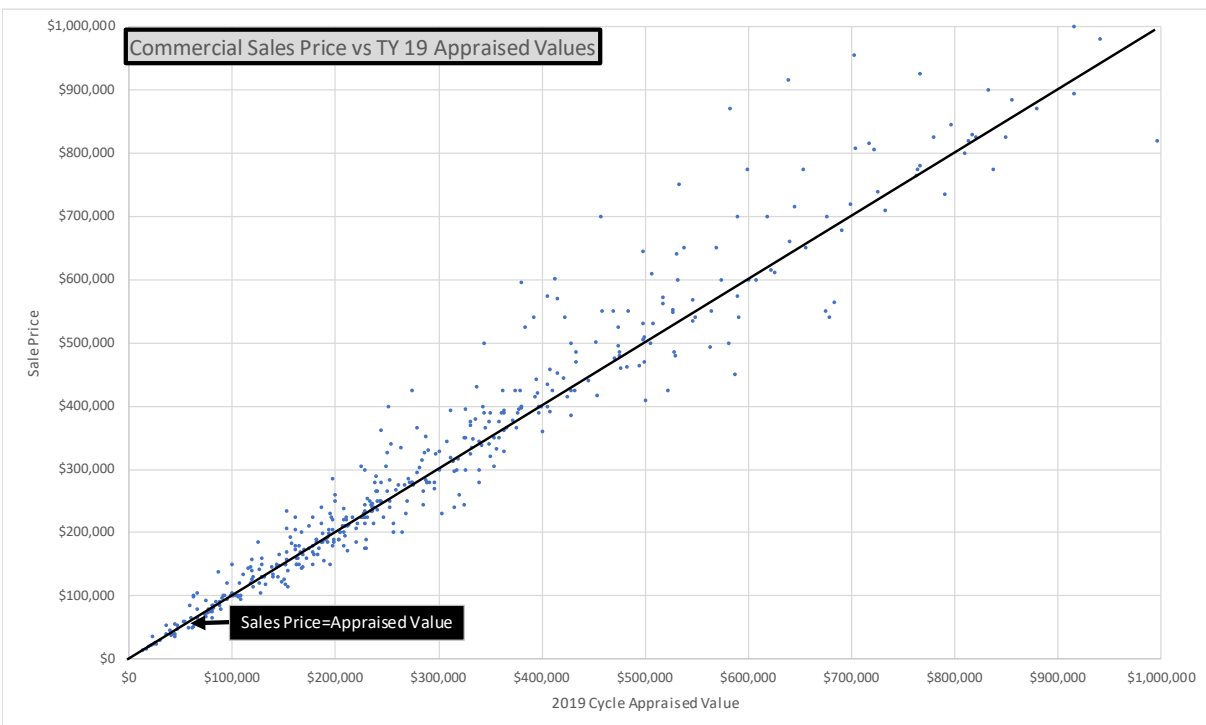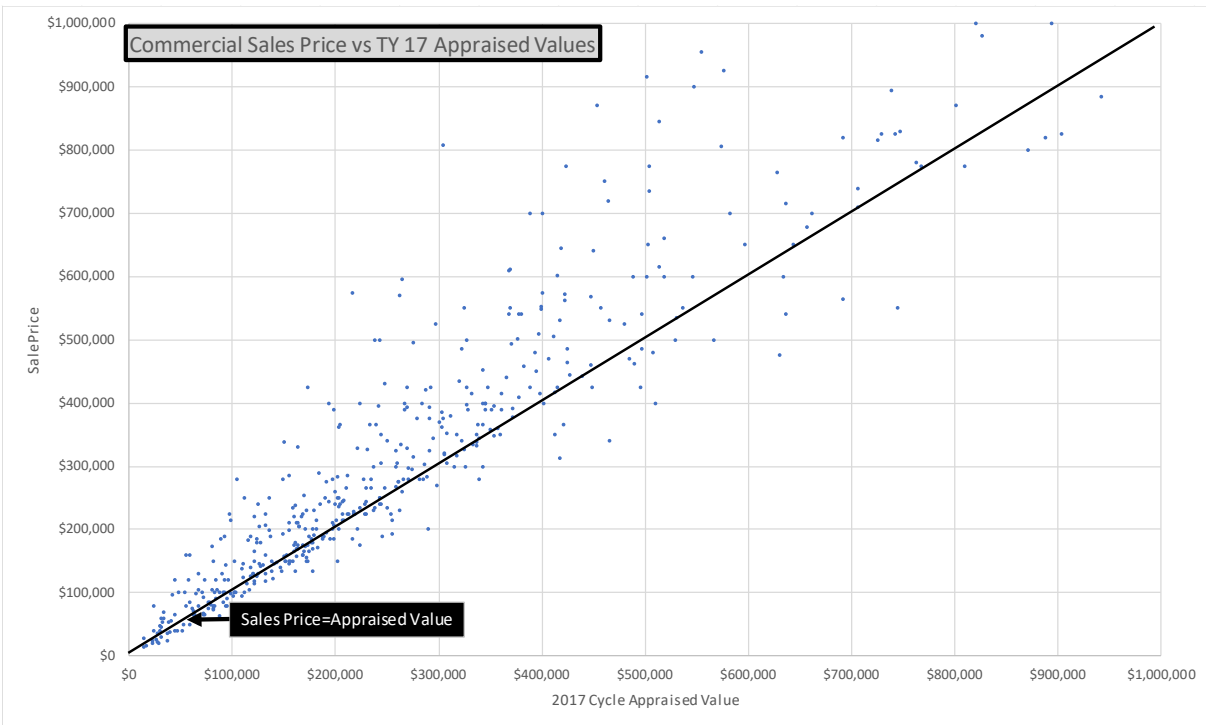
Region Analysis-Commercial

As with residential properties, the Department calculated the sales ratio statistics for the different administrative regions in the state. The following tables show the number of verified sales, statistics of central tendencies, and statistics concerning the distribution of the sales assessment ratios. All four regions have median assessment ratios, COD, and PRD values that are within the IAAO recommendation for a quality appraisal.

Regional Commercial Ratio Statistics TY17 Values and TY 19 Values								
	Region 1		Region 2		Region 3		Region 4	
Number of Sales	Previous	Current	Previous	Current	Previous	Current	Previous	Current
Total Observations	178	178	110	110	148	148	156	156
Used Observations	171	166	105	100	126	105	150	135
Measurement of Appraisal Levels								
Upper Bound	*96.00%*	*101.04%*	*97.81%*	*100.85%*	*91.27%*	*98.13%*	*80.85%*	*97.47%*
Median Ratio	**93.39%**	**99.72%**	**95.39%**	**99.46%**	**85.57%**	**94.29%**	**75.68%**	**95.01%**
Lower Bound	*88.30%*	*97.92%*	*91.93%*	*96.63%*	*79.71%*	*90.24%*	*69.72%*	*91.68%*
Upper Bound	*92.19%*	*100.85%*	*95.26%*	*102.23%*	*89.25%*	*95.60%*	*80.41%*	*96.44%*
Mean Ratio	**89.12%**	**98.73%**	**91.83%**	**99.87%**	**85.30%**	**92.85%**	**76.90%**	**94.05%**
Lower Bound	*86.05%*	*96.61%*	*88.40%*	*97.51%*	*81.35%*	*90.11%*	*73.39%*	*91.66%*
Upper Bound	*86.63%*	*99.61%*	*95.70%*	*102.40%*	*86.86%*	*95.61%*	*74.29%*	*92.39%*
Weighted Mean	**80.75%**	**94.53%**	**90.77%**	**99.16%**	**78.74%**	**89.41%**	**68.57%**	**88.18%**
Lower Bound	*74.86%*	*89.46%*	*85.84%*	*95.92%*	*70.62%*	*83.20%*	*62.84%*	*83.97%*
Measurement of Appraisal Uniformity								
Coefficient of Dispersion	16.747	9.819	13.535	8.583	20.452	11.950	23.509	11.232
Standard Deviation	20.340	13.851	17.733	11.871	22.389	14.197	21.779	14.017
Price Related Differentials	1.104	1.044	1.012	1.007	1.083	1.039	1.122	1.067

As the table shows, the ratios measuring the appraisal levels are all within the IAAO standards of 90% to 110% (Gloudemans, 1999). Similarly, the COD values are in the acceptable ranges in all four regions. Regions 3 and 4 had previous COD values that were outside of the IAAO recommendations indicating the need for new appraisal values (International Association of Assessing Officers, 2013). Although Region 2 is the only region with a PRD above the IAAO standard, all regions saw an improvement in the PRD, indicating less regressivity in the current appraisal cycle.

The following graphs show the distribution analysis of sales ratios for the four regions using the new appraisal values and the prior cycle appraisal values.

As the graphs show, the distributions all became more centralized, as can been seen in the height of the distribution's peak. Similar to the statewide graph, there does appear to be a shift to the left in all the regions, however, the shift is less dramatic relative to residential properties, with the possible exception of Region 4.

County Analysis-Commercial

There were six counties with at least 30 valid sales between October 1, 2017 and June 30, 2018. The following table shows the number of verified sales, statistics of central tendencies, and statistics concerning the distribution of the sales assessment ratios.

Select County Wide Commercial Ratio StatisticsTY17 Values and TY 19 Values												
	Current Cycle											
		Measurement of Appraisal Levels								Measurement of Appraisal Uniformity		
County	N	Lower Bound	Median Ratio	Upper Bound	Lower Bound	Mean Ratio	Upper Bound	Lower Bound	Weighted Mean	Upper Bound	Coefficient of Dispersion	Standard Deviation	Price Related Differentials
Flathead	46	93.69%	99.12%	102.05%	92.65%	96.77%	100.88%	89.02%	95.12%	101.21%	10.1484	13.8553	1.0174
Gallatin	64	89.33%	93.50%	97.69%	89.05%	92.29%	95.52%	82.19%	87.60%	93.01%	10.7214	12.9590	1.0535
Lewis And Clark	30	94.00%	99.46%	102.15%	92.26%	97.43%	102.59%	90.08%	97.03%	103.97%	9.9299	13.8224	1.0041
Missoula	56	97.63%	101.58%	105.79%	97.06%	100.73%	104.40%	85.39%	94.48%	103.57%	9.7452	13.7025	1.0661
Silver Bow	35	94.67%	97.63%	100.80%	92.82%	96.72%	100.61%	74.39%	87.71%	101.02%	8.0515	11.3313	1.1027
Yellowstone	62	91.70%	93.84%	97.69%	90.86%	93.91%	96.96%	81.62%	88.88%	96.13%	9.9899	12.0143	1.0567

The level of assessment was calculated for each of these counties. All the displayed counties have assessment levels (medians) that fall within the recommended range of 90%-110% (Gloudemans, 1999). Similarly, the PRD and the COD was also calculated for each county, and in all counties, the COD was inside of the recommended standards by IAAO (Gloudemans, 1999).

Valuation Method-Commercial

As a final check on the quality of the 2019 appraisal for commercial properties, it is helpful to examine sales ratio characteristics based on the method in which properties were appraised. The two approaches to valuing commercial property are an income-based approach and a cost-based approach.

Commercial Ratio Statistics TY 19 Values by Valuation Method		
Number of Sales	**Income**	**Cost**
Total Observations	419	172
Used Observations	375	130
Measurement of Appraisal Levels		
Upper Bound	*98.85%*	*100.13%*
Median Ratio	**97.92%**	**96.58%**
Lower Bound	*96.80%*	*94.48%*
Upper Bound	*98.19%*	*98.13%*
Mean Ratio	**96.83%**	**95.47%**
Lower Bound	*95.48%*	*92.81%*
Upper Bound	*96.31%*	*95.47%*
Weighted Mean	**93.25%**	**89.75%**
Lower Bound	*90.18%*	*84.04%*
Measurement of Appraisal Uniformity		
Coefficient of Dispersion	9.987	12.152
Standard Deviation	13.339	15.333
Price Related Differentials	1.038	1.064

As the table shows, none of the three measures of appraisal levels for the methods of valuation are statistically different from each other at a 95% confidence interval. This would indicate both methods of appraisal are valid for commercial sales. Similarly, both valuation methods have COD's within the acceptable IAAO range of values.

Conclusion-Commercial

Based on widely recognized norms and standards, the 2019 commercial appraisal is generally of high quality, as evidenced by this study. The goal of having a sample appraisal level within 10 percent of market value is met (International Association of Assessing Officers, 2013). The sample assessment level of 97.7 percent is within three percent of market value.

The reappraisal also meets uniformity standards, as evidenced by the coefficients of dispersion. The statewide COD of 10.5 is within the recommended range of 5.0 to 20.0 (International Association of Assessing Officers, 2013). The statewide PRD of 1.044 is outside the IAAO recommended limit of 1.03, indicating the possibility of some appraisal regressivity, but it is an improvement relative to the prior appraisal levels and most likely the result of a small sample size (International Association of Assessing Officers, 2013).

Statewide, the ratio distribution of the old and new values seems to indicate appreciation in most areas of the state, although generally not to the level seen in residential properties. Similar to residential properties, some areas appreciating more than others as evidence by the shift in median sales ratio from the old appraisal values to the new appraisal values.

Finally, the method that was used to appraise commercial properties does not seem to yield statistically different apparels levels as measured by the sales ratio and both methods are believed to be valid means of appraising commercial properties.

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