Using Confidence Intervals to Minimize Forecasting Error

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Motivation

In past years there have been numerous requests from the Legislature for standard error analysis of corporation income tax.
Select economic variables that model past revenue well.
Select economic variables that IHS consistently predicts well.
Place some level of certainty on a revenue estimate.
This will allow for comparison of models that use IHS economic forecasts as the main predictors.
Background:
FY 2013 = $2,078 million

FY 2013 General Fund Revenue by Major Source:

- Individual Income Tax: 50%
- Property Tax: 12%
- Corporation Tax: 8%
- Vehicle Taxes & Fees: 5%
- Oil Severance Tax: 5%
- Insurance Tax: 3%
- Remaining Sources: 14%
- Video Gaming Tax: 3%
Volatility of Corporation Tax

FY Corporation Income Tax Collections

$ Millions


$200 $180 $160 $140 $120 $100 $80 $60 $40 $20 $0
Sources of Forecasting Error

- Taxpayer Behavior
  - Montana law allows corporations to carry back current year losses for three years, and carry forward losses for up to seven years.

- Reliance on a limited number of large taxpayers.

- Random Error

- Inherent error of IHS variables used for modeling.
Corporation tax liabilities are divided into numerous sectors.
These sectors include, but are not limited to, mining, manufacturing, retail trade, and financial sectors.
IHS economic variables are used to model each sector individually.
Sector estimates are combined to form a final revenue estimate for corporation tax liability.
Study Sector-by-Sector Error

\[ \varepsilon \ (\text{error}) = \frac{\text{actual value} - \text{estimate}}{\text{actual value}} \]
Example: Manufacturing Sector

CY Manufacturing Sector Corporation Tax Liability

$ Millions

WTI Price Fits Historical Data Well...
...but how well do IHS forecasts of WTI predict manufacturing tax liability?
Actualls vs. Estimates

<table>
<thead>
<tr>
<th>CY</th>
<th>First Year</th>
<th>Second Year</th>
<th>Third Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>50%</td>
<td>50%</td>
<td>N/A</td>
</tr>
<tr>
<td>2005</td>
<td>45%</td>
<td>67%</td>
<td>70%</td>
</tr>
<tr>
<td>2006</td>
<td>29%</td>
<td>57%</td>
<td>73%</td>
</tr>
<tr>
<td>2007</td>
<td>33%</td>
<td>48%</td>
<td>70%</td>
</tr>
<tr>
<td>2008</td>
<td>-5%</td>
<td>13%</td>
<td>49%</td>
</tr>
<tr>
<td>2009</td>
<td>1%</td>
<td>-44%</td>
<td>-40%</td>
</tr>
<tr>
<td>2010</td>
<td>-6%</td>
<td>3%</td>
<td>-17%</td>
</tr>
<tr>
<td>2011</td>
<td>4%</td>
<td>11%</td>
<td>3%</td>
</tr>
</tbody>
</table>
A confidence interval for the individual errors would require that their distribution be known.

With such a small sample, confidence in the true distribution is small.
Error Summary and Distribution

Summary Statistics of Manufacturing Tax Liability Error Term by Estimate Year

<table>
<thead>
<tr>
<th>Statistic</th>
<th>First Year</th>
<th>Second Year</th>
<th>Third Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>19%</td>
<td>26%</td>
<td>30%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>23%</td>
<td>37%</td>
<td>47%</td>
</tr>
</tbody>
</table>

Histogram of Manufacturing First Year Estimates' Error Terms
Bootstrap Sample Mean

<table>
<thead>
<tr>
<th>Statistic</th>
<th>First Year</th>
<th>Second Year</th>
<th>Third Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Error</td>
<td>19%</td>
<td>26%</td>
<td>30%</td>
</tr>
<tr>
<td>Standard Error</td>
<td>8%</td>
<td>12%</td>
<td>16%</td>
</tr>
</tbody>
</table>

Histogram of First Year Estimates' Mean Error Terms
## Aggregate Results

### CY Corporation Income Tax Liability

#### 95% Confidence Intervals for the Aggregate Average Error Term of the Sector-Based Estimate ($ Millions)

<table>
<thead>
<tr>
<th></th>
<th>Mean Error</th>
<th>t-Statistic</th>
<th>Standard Error</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Year Error Bound</td>
<td>$15.1</td>
<td>± 2.365</td>
<td>× 6.7</td>
<td>[-$0.8,$30.9]</td>
</tr>
<tr>
<td>Second Year Error Bound</td>
<td>$13.3</td>
<td>± 2.450</td>
<td>× 9.3</td>
<td>[-$9.6,$36.1]</td>
</tr>
<tr>
<td>Third Year Error Bound</td>
<td>$10.2</td>
<td>± 2.571</td>
<td>× 12.2</td>
<td>[-$21.2,$41.6]</td>
</tr>
</tbody>
</table>

### CY Corporation Income Tax Liability

#### 95% Confidence Intervals for the Sector-Based Estimate ($ Millions)

<table>
<thead>
<tr>
<th>Estimate Year</th>
<th>Estimate</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>% Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>2015</td>
<td>$135.0</td>
<td>$134.2</td>
<td>$165.9</td>
<td>24%</td>
</tr>
<tr>
<td>2016</td>
<td>$139.2</td>
<td>$129.6</td>
<td>$175.3</td>
<td>33%</td>
</tr>
<tr>
<td>2017</td>
<td>$145.3</td>
<td>$124.1</td>
<td>$186.9</td>
<td>43%</td>
</tr>
</tbody>
</table>
Fiscal Year Results

### FY Corporation Income Tax Liability

Using 95% Confidence Intervals for the Sector-Based Estimate

<table>
<thead>
<tr>
<th>Estimate Year</th>
<th>Estimate ($ Millions)</th>
<th>Lower Bound ($ Millions)</th>
<th>Upper Bound ($ Millions)</th>
<th>% Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>2015</td>
<td>$149.7</td>
<td>$149.0</td>
<td>$177.7</td>
<td>19%</td>
</tr>
<tr>
<td>2016</td>
<td>$147.9</td>
<td>$147.2</td>
<td>$176.2</td>
<td>20%</td>
</tr>
<tr>
<td>2017</td>
<td>$151.7</td>
<td>$142.9</td>
<td>$184.8</td>
<td>28%</td>
</tr>
</tbody>
</table>

### FY Corporation Tax Liability Using 95% Confidence Intervals

![Graph showing FY Corporation Tax Liability using 95% Confidence Intervals]
Are the Variables Independent?

FY Corporation Income Tax Liability
95% Confidence Intervals for the Sector-Based Estimate Assuming Partial Dependence
($ Millions)

<table>
<thead>
<tr>
<th>Estimate Year</th>
<th>Estimate</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>% Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>2015</td>
<td>$149.7</td>
<td>$142.0</td>
<td>$184.7</td>
<td>29%</td>
</tr>
<tr>
<td>2016</td>
<td>$147.9</td>
<td>$140.0</td>
<td>$183.3</td>
<td>29%</td>
</tr>
<tr>
<td>2017</td>
<td>$151.7</td>
<td>$133.9</td>
<td>$193.9</td>
<td>40%</td>
</tr>
</tbody>
</table>

FY Corporation Tax Liability with 95% Confidence Interval
Results

- Adjusted corporation sector liability models to minimize future error bounds.
- Produced an aggregate estimate and corresponding error bounds for the sector-specific model.
- Calculated error bounds associated with alternative models.
- Allows for comparison of forecasts that use economic forecasts as the main predictors.
Sector-Based Model

FY Corporation Tax Liability Using 95% Confidence Intervals

- Sector-Based Model
- Upper Bound
- Lower Bound
Single Variable Model: WTI

FY Corporation Tax Liability with 95% Confidence Interval

- Single Variable (WTI) Model
- Upper Bound
- Lower Bound
Single Variable Model: Personal Income

FY Corporation Tax Liability with 95% Confidence Interval

- Single Variable (Personal Income) Model
- Upper Bound
- Lower Bound
Single Variable Model: US Corporate Profits

FY Corporation Tax Liability with 95% Confidence Interval

$ Millions

2012 2013 2014 2015 2016 2017

Single Variable (U.S. Corp. Profits) Model
Upper Bound
Lower Bound
Lagged Corporate Profits Model

FY Corporation Tax Liability with 95% Confidence Interval

- Single Variable (U.S. Corp. Profits) with Lags Model
- Upper Bound
- Lower Bound
Future Work

- Continue peer-review process.
  - Statewide Economists
  - Past professors
  - Past colleagues
  - PEW center for the states

- Incorporate relevant suggestions.

- Apply methodology to other large or volatile revenue sources.