



MONTANA LEGISLATIVE BRANCH

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Director
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DATE: March 12, 2018

TO: Legislative Finance Committee

FROM: Sam Schaefer

RE: Statistical Analysis of Variability of General Fund Revenue in Montana

With the passage of [SB 261](#) the Legislative Fiscal Division (LFD) was tasked with researching rainy day funds (RDF) and analyzing Montana's volatility to come up with an appropriate RDF size. Using part of the methodology outlined in "Saving for a Rainy Day: Estimating the Appropriate Size of U.S. State Budget Stabilization Funds" (Zhao, 2014), a working paper from the Federal Reserve Bank of Boston, RDF sizes based on Montana's historical revenue collections were developed with corresponding levels of risk.

The methodology described in Zhao's paper attempts to model a state's long-term revenue trend, which will provide a means to model the short-term component as well. Ultimately, the characteristics of the short-term component (revenues above or below the long-term trend) can be used to assign varying levels of risk to RDF sizes. In theory, reserve funds should be used when revenues in a particular year are below the long-term revenue trend. Zhao's paper looks at modeling revenues using personal income, personal income squared, and a time-trended model. In addition, the long-term trend was also modeled using a Hodrick-Prescott (HP) filter. This filter is a mathematical tool that is commonly used to remove cyclical components of a time series from the data. Given its nature, the HP filter is often used to model business cycles.

Since Montana uses a biennial budget, biennial revenues dating back to 1972 were used and are shown in Figure 1 below. For reasons that will be explained later, amounts are measured in units of 2017 dollars.

Inflation-Adjusted General Fund Biennial Revenues & Modeled Trends

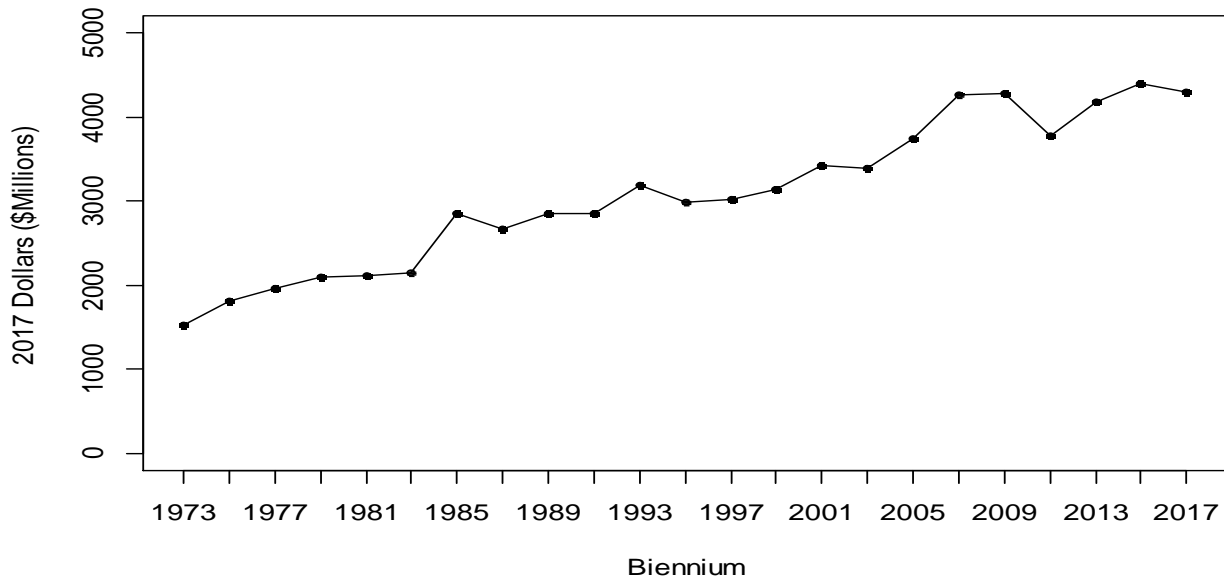


Figure 1

Figure 2 displays the biennial revenues along with the modeled underlying long-term trends.

Inflation-Adjusted General Fund Biennial Revenues & Modeled Trends

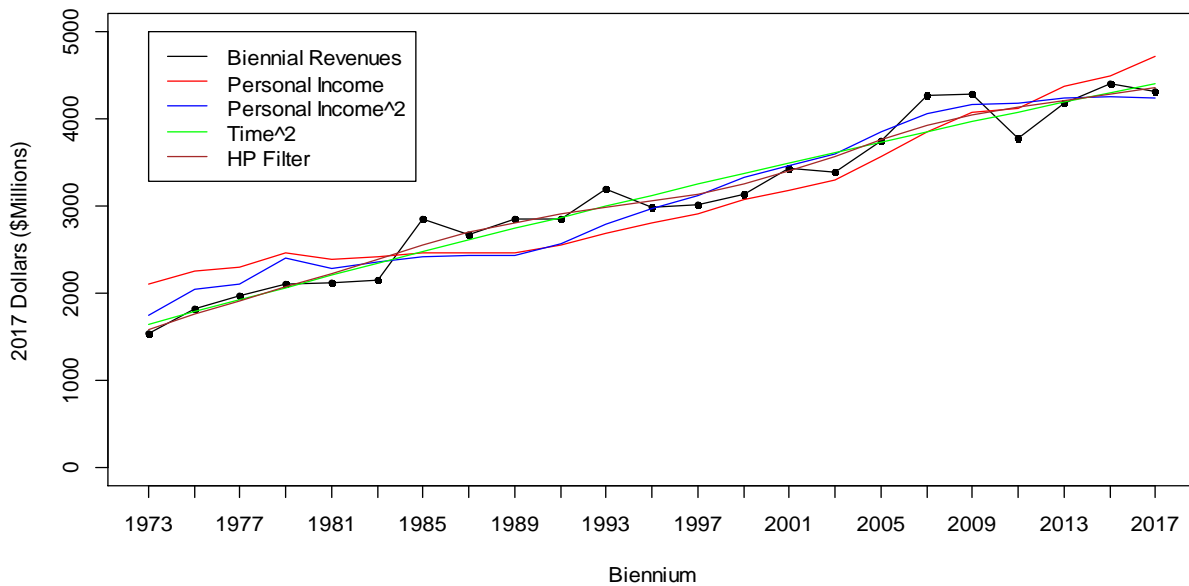


Figure 2

The idea behind the personal income squared model is that as personal income becomes increasingly large, the effect on revenues may increase, reflecting the progressive nature of tax systems. Model diagnostics suggest that the personal income squared and time squared models are more reliable fits and also satisfy linear regression assumptions. As a result, these models, along with the HP filter, were used for the rest of this analysis. The difference between the long-

term trend and actual revenue is known as the residual, and will be regarded as the short-term revenue component. The short-term component for both models is shown below as well as the cyclical component from the HP filter.

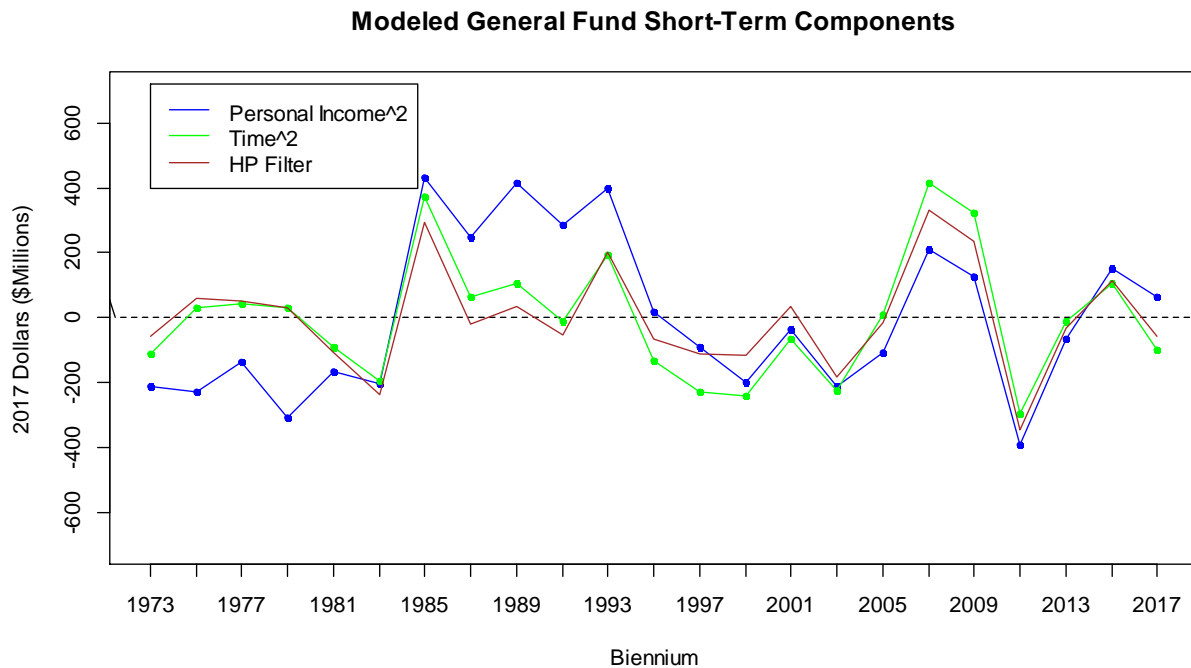


Figure 3

Table 1 below displays the largest drop between the short-term component and the long-term trend as well as the residuals at the lower 5th, 10th, 20th, and 25th percentiles. Note that by using units of 2017 dollars the short-term component from the 1970's are able to be compared directly to the most recent short-term components. Percentiles were estimated using linear interpolation.

Short-Term Component Lower Percentiles					
	Maximum Departure	5%	10%	20%	25%
Personal Income Squared	392	380	284	210	205
Time Squared	294	286	236	206	148
HP Filter	345	329	219	113	108

Table 1

The income and time squared models produce varying results regarding appropriate RDF sizes. For instance, to weather the largest recession the personal income squared model suggests that the state would need to set aside \$392 million while the time squared model suggests \$294 million. Note that for both methods this occurred in the 2011 biennium. Furthermore, the personal income squared model suggests that \$380 million would be enough to handle 95% of recessions while the time squared model suggests only \$286 million is needed. Since both models have relatively strong linear fits the question arises if a simple time trend or income trend is more appropriate to explain underlying long-term tax trends. The HP filter provides values in between the income squared and time squared trends for the maximum departure and 5th percentile. However, the filter's short-term components decrease dramatically at the 20th and 25th percentiles.

Instead of analyzing the historical short-term component directly, methodologies exist to forecast revenues using long-term time trends with corresponding prediction intervals. These intervals can essentially serve as the risk levels shown above in Table 1. Figure 4 shows the non-inflation adjusted biennial revenue collections.

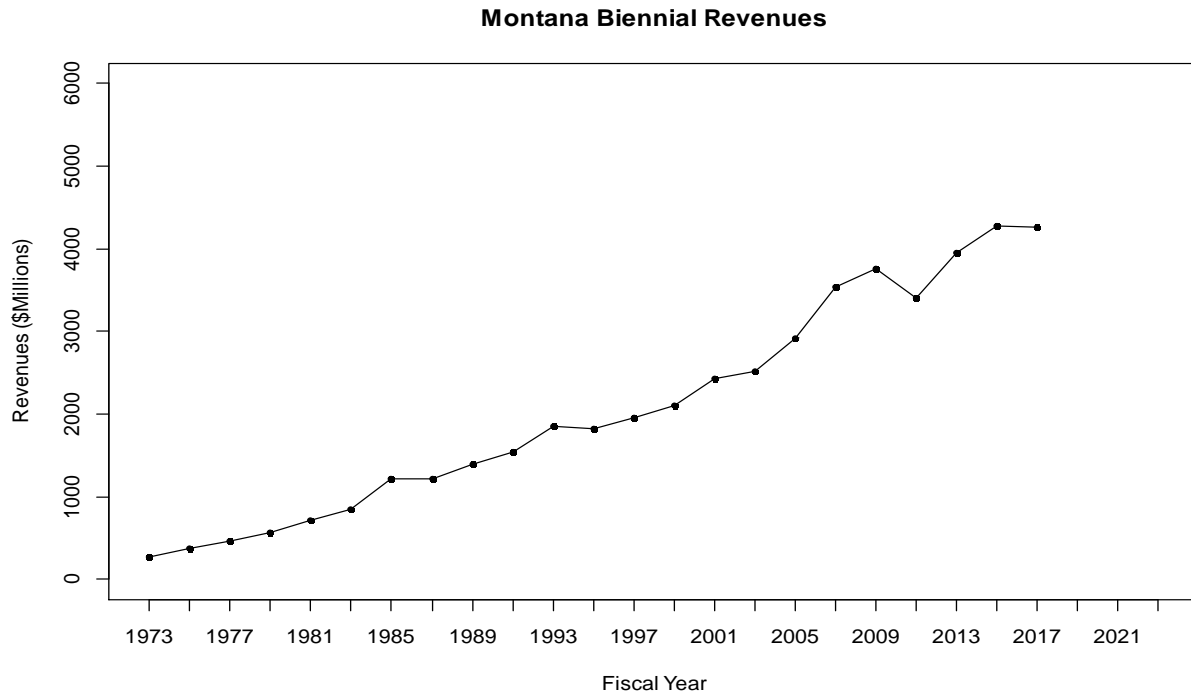


Figure 4

To calculate theoretical risk bounds moving forward, the future point must be known with certainty. While variables such as personal income would need to be estimated, a variable such as the year representing time is known with certainty. A linear trend modeling Montana's revenue pattern using only time as a predictor is shown below in Figure 5.

Montana Biennial Revenues with Linear Trend

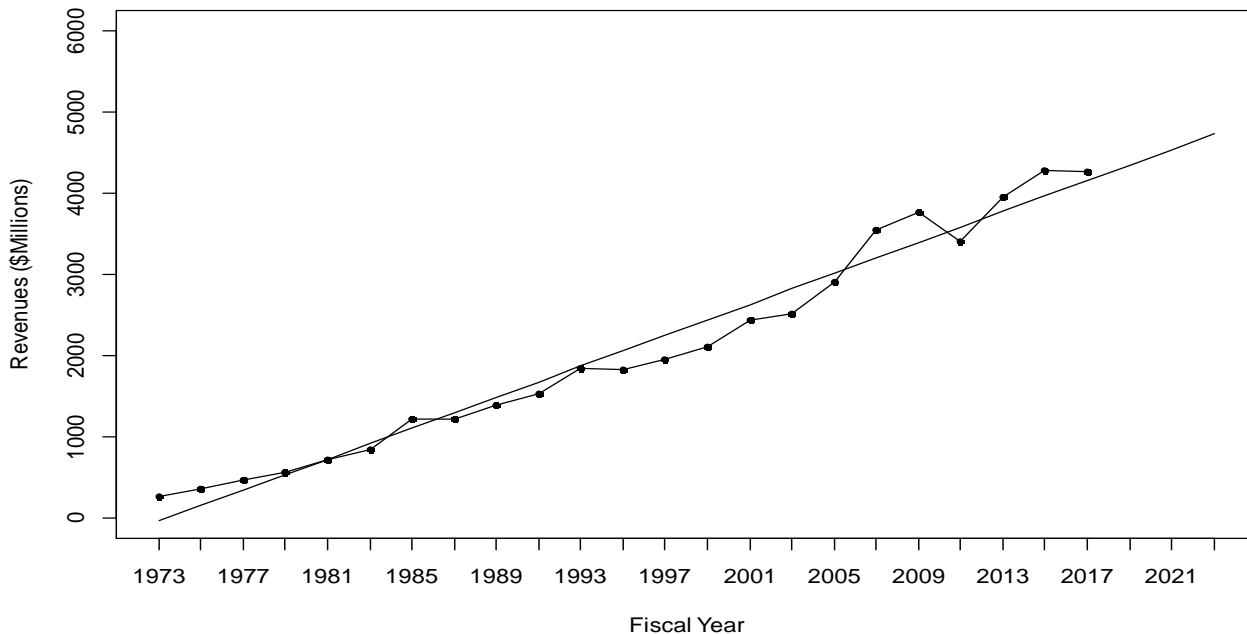


Figure 5

The corresponding prediction intervals (risk intervals) are shown below in Figure 6.

Montana Biennial Revenues with Linear Trend

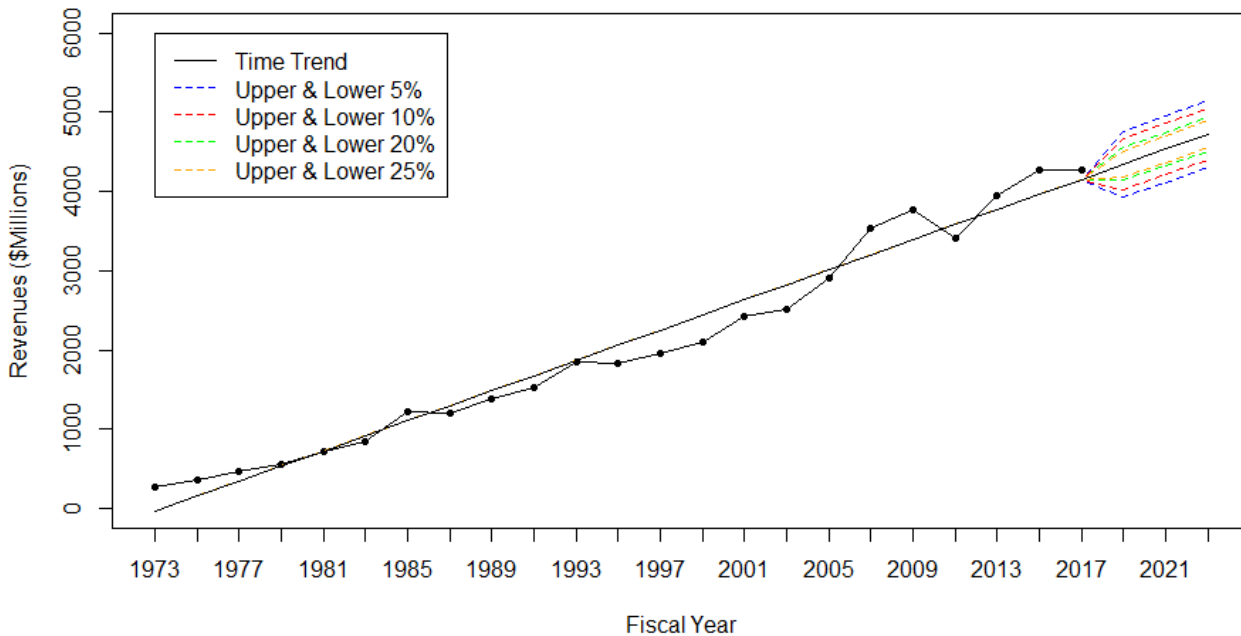


Figure 6

Similarly to earlier, a time squared component can also be included in the model to account for any increased responsiveness of revenues to time as time increases. The new modeled trend and prediction intervals are shown below in Figures 7 and 8.

Montana Biennial Revenues with Quadratic Trend

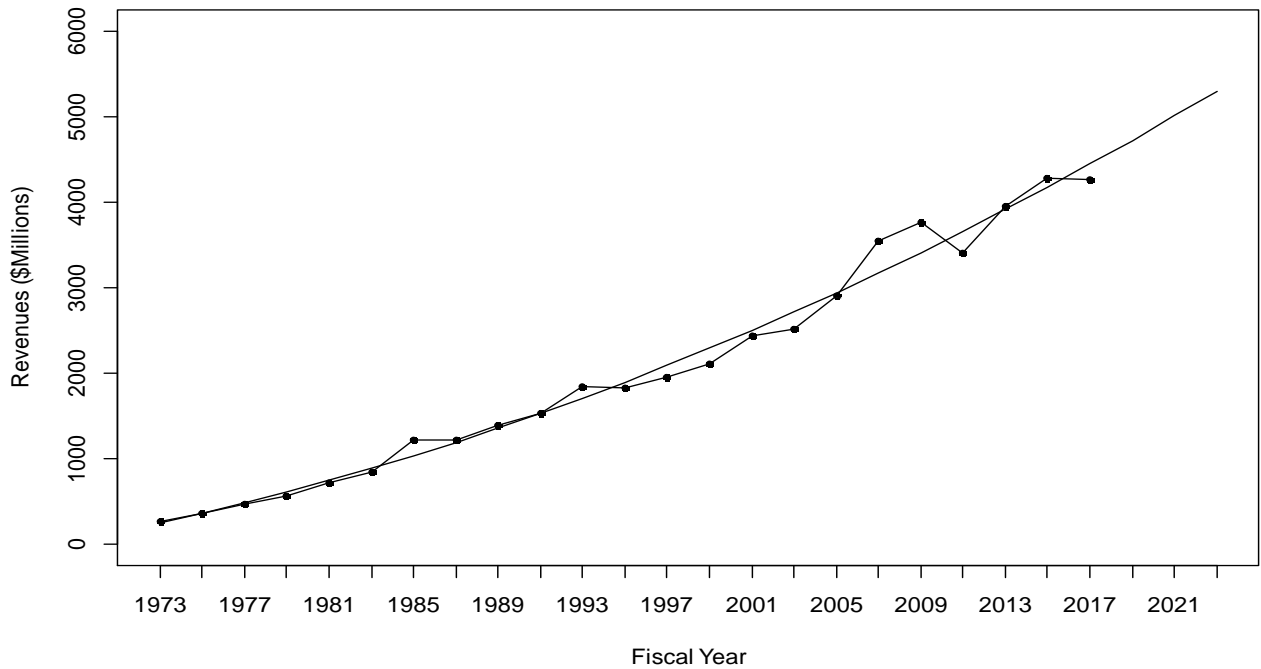


Figure 7

Montana Biennial Revenues with Quadratic Trend

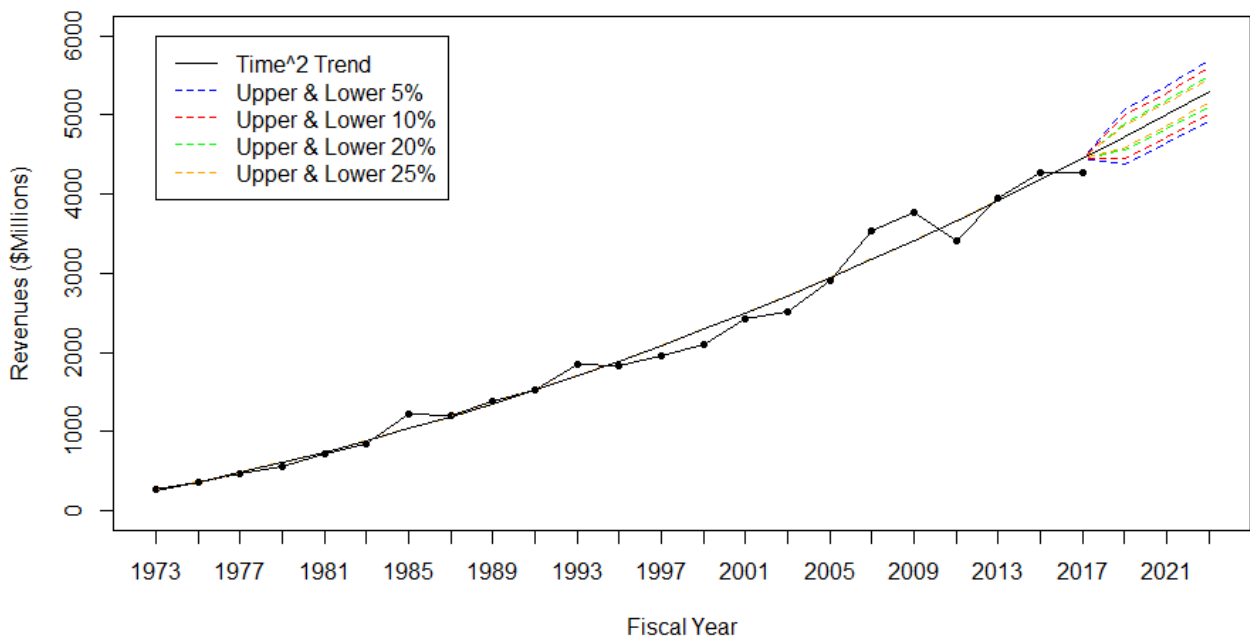


Figure 8

Table 2 below shows the values associated with varying levels of risk, similar to those seen in Table 1 except for the forward-looking methodology.

Forward-Looking Risk Levels				
	5%	10%	20%	25%
Time	414	319	207	165
Time Squared	339	260	169	135

Table 2

This method does not produce a maximum RDF, as it is forward looking and has no one recession point to compare to. Table 3 below shows calculated results for the various methodologies employed in this paper.

RDF Size Required					
		5%	10%	20%	25%
Personal Income Squared	(Backward Looking)	380	284	210	205
Time Squared	(Backward Looking)	286	236	206	148
Time	(Forward Looking)	414	319	207	165
Time Squared	(Forward Looking)	339	260	169	135
HP Filter		329	219	113	108

Table 3

As the table shows, the differing methods yield varying results. The ultimate question is how to best model the underlying historical trend, and if this trend will continue in future years. This is a difficult question, and answers vary from study to study. Also, the question arises if any one method may prove to be more useful than others in relation to Montana's specific RDF rules.

Looking back at Figure 2, the personal income squared trend has more fluctuation than one would desire to represent and underlying trend. This is likely due to the inherent cyclical nature of personal income in itself. Using a metric such as time may be more appropriate. Model diagnostics along with a simple glance at Figures 5 and 7 suggest that using the time squared trend is superior to the trend using only time. This leaves the two time squared techniques along with the HP Filter as the remaining options.

If the sole purpose of the remaining methodologies was to purely estimate reserve sizes it would be difficult to argue for any one over the other two. If only one were to be chosen, examining overall usefulness could be a reasonable step in the decision-making process. While the retrospective time squared model and HP filter perform well in estimating an underlying long-term trend (Figure 3), they don't provide the same forecasting power as the forward-looking time squared model. The HP filter operates in a closed data set, and isn't intended to be used for future predictions. However, the forward-looking time squared model provides a means for a quick check on revenue estimates, can be updated quickly, and operates in real-time (no conversions to current year dollars are required). Finally, results from this method in Table 3 tend to be in the middle of the five methodologies' results. Table 4 below shows this method's results.

RDF Size Required for Forward Looking Time-Squared Trend					
Risk Levels		5%	10%	20%	25%
Reserve Amount (\$ Millions)		339	260	169	135
Percent of \$2.3 Billion Annual Budget		15.1%	11.6%	7.5%	6.0%
Percent of \$4.5 Billion Biennial Budget		7.5%	5.8%	3.8%	3.0%

Table 4

Overall, these results are consistent with what Montana-specific literature review is available. The purpose of this study is to simply assign some level of risk to dropping below historical trends, while also trying to best model that underlying trend. Ultimately, to put these results into practice, the legislature and executive would need to establish a level of risk they are willing to undertake. In other words, at what level of a revenue shortfall would a special session take place, compared to spending out of a reserve fund? Once this has been established, these results can begin to be set in motion in the context of the rules set forth in SB 261.